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Supply chain models for an ethanol and distillers grains producer and a distillers grains-based feed producer

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ABSTRACT

In recent years, there have been substantial interests in the joint production of ethanol and distillers grains (E&DG) from corn. At the same time, there have been corresponding increases in the production of animal feed based on distillers grains (DG). Under such circumstances, ethanol and DG are produced by an E&DG producer, and DG serves as input to feed production by a feed producer.The objective of this paper is to study the strategies of both producers in different models in order to maximize their own profit with more ethanol produced by consuming more DG in the feed market.

First, we investigate the economic relationships such as pricing between the E&DG producer and the DG-based feed (feed) producer under Stackelberg competition as well as under coordination when both producers have their own linear production costs, respectively. (1) Specifically, for competition, we construct a Stackelberg model with an E&DG producer as the leader and a feed producer as the follower, and examine the consequences in terms of profits, prices, and production and purchase quantities. As the DG fraction increases, the E&DG producer loses or gains more profit than the feed producer under different conditions. Under specific condition, as the DG fraction increases, both producers have higher profit with the higher quantity of DG as well as ethanol so as to help the increasing ethanol market. (2) For coordination, we consider a centrally coordinated model where producers are viewed as one group. Compared with the Stackelberg model, the centrally coordinated model has higher total profit to be shared by both producers as they are optimizing the total profit as a single company with more DG as well as ethanol produced.

Second, this paper extends Stackelberg model with a quadratic unit joint production cost for the E&DG producer. For this model, we investigate the economic relationships such as pricing, profit for both producers and compared with the euqilibrium solution in the Stackelberg model with a linear joint production cost for the E&DG producer.

Key Words: Game theory; Supply chain; Ethanol; Distillers grains; DG-based feed

1. INTRODUCTION

As the renewable energy consumption in the U.S.A. grows, ethanol, a renewable source of energy as supplement for gasoline, is increasing by the stimulation. Nowadays, ethanol is the only renewable motor fuel produced in large quantity, i.e., the United States produced 3.9 billion gallons of ethanol in 2005, up from 3.4 billion gallons in 2004. In addition, most ethanol is produced from corn. For instance, approximately 97% of ethanol in the U.S.A. is produced from corn. The corn-based ethanol industry is poised to significantly contribute to meet rising energy demands in the coming years.

1.1 Introduction to the corn-ethanol industry

In 2006, there were 102 ethanol bio-refineries with another 42 under construction, up from 50 ethanol plants in 1999. Iowa and other states in Midwest America form the traditional corn zone and this area is quickly becoming the major area for corn-ethanol production. Dry mill processing of corn results in two products—ethanol and distillers grains (DG). Approximately 1/3 ton of each of the constituent products are produced from 1 ton of corn processed [1]. As a renewable energy source, ethanol is becoming more available for automobiles around the United States.

The ethanol industry has changed the shape and structure of the corn industry in recent years. More policies from the federal and some states are keeping stimulating the ethanol industry to expand. The implementation of the Clean Air Act in 1990 helped propel renewable energy. In several states, there is a 10% ethanol mandate, e.g., all gasoline sold in the state of Minnesota (1997), Hawaii (2006), Montana (2006), Missouri (2008), and Washington (2009) must contain at least 10% ethanol. In addition, the EPAct has several important incentive provisions. The most widely publicized provision of the EPAct, the Renewable Fuel Standard (RFS), applies to corn ethanol. Implementation of the RFS began in 2006 with 4 billion gallons per year, and is predicted to increase to 7.5 billion gallons per year by 2012 [2]. Currently, the subsidy policy to the bio-fuel, mixing ethanol with gasoline, is volumetric ethanol excise tax credit (VEETC).

The incentives from the states play an important role for the expansion of the local ethanol industry. For example, Minnesota has an incentive program of 20 cents per gallon on up to 15 million gallon of ethanol per year for maximum of 10 year; and Wisconsin has the similar policy of ethanol production incentive to corn-ethanol producers with 20 cents per gallon produced.

As we know, several outputs that emerged from a single productive activity are the fundamental economic situation. More and more DG are produced as the expansion of the corn-ethanol industry, since it is one of the joint products in corn-ethanol production. Generally, DG is sold to the feed producer or local livestock producer as a protein source for beef, swine, cattle, poultry, and so on [3], e.g., Land O'Lakes Purina Feed LLC purchases DG for their feed manufacturing [4].

Hawkeye Energy holding LLC, an important corn-ethanol producer in the state of Iowa, sells DG to local livestock producers and interacts with feed companies on a daily basis around each of its ethanol plants. Behnke [5] mentions that the volume available and the relative price of DG have forced many feed producers to using greater levels than before. "In the past, the ethanol producers wanted to 'make the mash go away' and livestock producers said they would use it only because it is free. Today, ethanol producers see it as a viable and valuable co-product and livestock producers determine how to best utilize it in their ingredient feed mix" [6].

From a survey, DG is shipped to local (51%), state (33%) , export (14%) as the ingredient in the feed, and other (2%) as the industrial use. DG is used in livestock rations as the protein supplement [7], and less than 30 to 40% of the ration dry material (DM) as DG can be fed to some dairy cattle or around 40 to 50% can be included in the diets of finishing cattle. When feeding more than 20% DG, the livestock producer is likely to feed excess protein. To counter this, forages consisting mostly of corn silage and excess phosphorus are a consideration [8],[9].

Some researches show that the U.S. livestock feed demand for DG can accommodate the rapid growth in DG production. Also according to the responses from 10 ethanol producers, i.e. ADM, Hawkeye, ACE, etc., all DG is sold out without surplus nowadays. In 2008, 23M tons of DG was produced in U.S.A. From the record in Oct. 22, 2009, the capacity of ethanol in USA is 13131.4 million gallon per year (39.8M tons per year). Therefore, the capacity of DG is almost 39.8M tons/year by following the 1:1 fixed proportional rate between ethanol and DG.

However, with the rapid expansion of the ethanol industry, the production of DG as byproduct also keeps the rapid growth. According to the Energy Independent and Security Act of 2007, the renewable fuels standard (FRS) requires 36billion gallons of ethanol=109M tons in 2022. So, the capacity of DG will be at least 109M tons/year. Although the feedlot farmer can consume more DG with the higher inclusion rate in the feed, and there exists the potential growth export market, it will be a big problem in the next decade that the supply of DG from the ethanol producers is becoming greater than the demand from the feed producers.

All ethanol producers have to face the problem how to deal with the excess distillers grains. Therefore, it is important to propose some strategies for the corn-ethanol supply chain to overcome the potential challenge in the next several years.

Based on the industry, what this paper depicts is a two-producer vertical channel. An upstream producer produces two outputs from the same process, and a downstream producer utilizes only one of the two outputs as intermediate material in her own production. In this paper, the upstream producer is the producer of ethanol and DG (the E&DG producer), who produces ethanol and DG from corn, and then sells ethanol to the ethanol-based fuel market, e.g., ADM (Archer Daniels Midland). AMD has sales from the bio-products segment of about $3.59 billion in 2008, up from $3.06 billion in 2007. The downstream producer is the DG-based feed producer (the feed producer), who purchases DG as the ingredient for feed production, e.g., Land O'Lakes, who has the sales from the feed segment of about $3.9 billion in 2008, up from $3.1 billion in 2007.

The price the E&DG producer sets and the quantity of ethanol this producer produces do not have any effect on the market price. Each is small, relative to the ethanol market. Therefore, the E&DG producer does not need to worry about what price to set for ethanol. Instead, E&DG producer is concerned with only how much to produce [10]. Nowadays, in the U.S.A. ethanol market, there are more than 100 ethanol producers, and none has a dominant market power in the ethanol market [7],[11],[12]. Bole and Londo [13] mentioned the bio-fuel industry is mostly a price-taker, because the price of ethanol is not promptly followed by an increase in the feedstock price.

In recent years, expansion of the ethanol industry has attracted substantial research attention. The production quantities of ethanol, DG, and DG-based feed also increase.

However, corn is main ingredient in feed, and its price is a fluctuant factor. Under such circumstances, we investigate the impact from the change in the DG fraction on the ethanol market when the other ingredients' cost increases as the corn price increases. The objective is to find under some specific conditions, the increase of the DG fraction can increase the amount of DG as well as ethanol for helping the expanding ethanol market when the other ingredients' cost increases. In addition, we investigate the competitive and the coordinated relationship for the E&DG producers and the DG-based feed producers. The objective of this paper is to gain insight from the competition and coordination between these two producers with the joint production, which one is helpful to the quick expanding ethanol market, and how both producers can be better off. Since the rapid growth of DG is caused by the expanding corn ethanol industry, how the huge amount of DG can be solved by the comparison between the competition and coordination.

Most of the studies focus on the measurement of the market structure and power for a single product, and ignore the joint products in a vertical structure supply chain. Alternatively, the joint production cost is allocated to determine the price for each output. In this paper, two producers in the game competition are—one is operating the joint production without cost allocation for the price making, the other is operating processing production. The producer operating the joint production is the price-taker for one product and the price-maker for another product.

Motivated by this, we address the following questions in this paper: What is the equilibrium solution for the joint production? What is the condition under which the increase of the DG fraction can increase the amount of DG as well as ethanol and the profit of both producers? What is the economical impact under the Stackelberg model after comparing with the centrally coordinated (CC) model? In such a case, what is the supply chain contract for pursuing channel coordination? Moreover, what is the impact from the price of one product given by the market?

1.2 Literature Review

Mathematical methodology provides relative techniques for economic analysis. In the vertical structure with joint products, the tasks are to solve the equilibrium solution, to determine the optimal solution in the centrally coordination, and to establish the contract for the coordination. Consequently, we reviewed papers related to game theoretical models of successive monopolists' supply chain.

Relative to the big ethanol market, the E&DG producer is small and is concerned with only how much to produce [7], i.e., in the U.S.A., all ethanol producers have not a dominant market power in the ethanol market [5].

As for the game with the joint production, Elishberg and Steinberg [14],[15] modeled joint production-marketing strategies for two firms with asymmetric production cost structures in Stackelberg competition, where the producer is the leader and the distributor is the follower. Baumgartner [16] established a similar configuration as our paper, by exploring the price ambivalence of secondary resources on a vertically integrated two-sector economy.

As Cheng and Liao [17] mentioned, many producers involved with chemical, petroleum production, and meat packing production use the cost-plus pricing approach to determine the selling price for products. In this situation, joint costs are allocated before the determination of the joint product cost-plus price. In this paper, however, the cost allocation is not implemented into the model.

Tirole [18] investigated the concept of vertical supply chain with double marginalization in the industrial organization. Double marginalization refers to the loss of profits and higher retail price in a decentralized supply chain because of two successive mark-ups. Double marginalization occurs because the retailer does not consider the producer's profit while setting the retail price. Weyl [19] described the Spengler-Stackelberg industrial organization. There is a common additional assumption that upstream chooses its prices before downstream in the spirit of von Stackelberg. Jeuland and Shugan [20] and Irmen [21] studied the absolute output margin as the decision variables under the Cournot scenario. Jeuland and Shugan [20] investigated the double marginalization in several scenarios under general conditions. They showed that a channel of distribution consists of different channel members, each having their own decision variables.

In addition, Young [22] studied that two firms make pricing decisions simultaneously by their own output margin to reach a Nash Equilibrium. Lee and Staelin [23] investigated different supply chain structures and power scenarios in two echelon supply chains such as the producer-leader and retailer-leader, etc. They found the type of vertical strategic interaction (VSI), as defined by the slope of the followers' response function, is the driving force behind equilibrium decisions on supply chain leadership and pricing. In addition, they demonstrated the linear demand function is not a necessary condition for any type of VSI and suggested the linear-nonlinear dichotomy is not important for robustness of the analytical results. Lau and Lau [24] compared different possible gaming processes in two-echelon vertical supply chain consisting of a producer and a retailer. They discussed the producer being the leader, the retailer being a leader by declaring a dollar output margin, and the retailer being the leader by declaring a percentage margin.

Yang and Zhou [25] considered pricing and quantity decisions of a two-echelon system with a producer who supplies a single product to two competitive retailers. They analyzed the effects of the duopolistic retailers' different competitive behavior: Cournot, Collusion and Stackelberg. The comparison of the equilibrium solutions for these three two-echelon models is made. Choi [26] studied three non-cooperative games of different power structures between the two producers and the retailer, i.e., two Stackelberg games between each of the producers and the retailer, and one Nash game.

Bard et al. [27] and Rozakis et al. [28] studied the Stackelberg model in which the government has the leadership in the corn-ethanol production.Tyrchniewicz [29] figured out that the supply chain leadership in the corn-ethanol industry usually is holdby the E&DG producer.

Compared to the competition, Jeuland and Shugan [20] explored the problems inherent in channel coordination, and addressed some questions. The structure of the simple model is directly usable to analyze the case of a franchise in which a producer sells locally one product through one retailer. Savaskan and Bhattacharya, etc. [30] also presented the case with the centrally coordinated system as the benchmark scenario to compare the decentralized models with respect to the supply chain profits and the performance.

Frohlich [31] pointed out, in practice, an integration of several producers is difficult to achieve, in spite of knowing the theoretical benefits of supply chain integration for years. In the real world, the E&DG producer and the feed producer are separate from their own business.

Cachon and Lariviere [32] studied the supply chain coordination with a revenue sharing contract. This type of contract is prevalent in the videocassette rental industry relative to the price contract. Giannoccaro and Pontrandolfo [33] proposed a coordinated model with three-stage supply chain, based on the revenue sharing contract. So far, different contract models have been developed, including the revenue sharing contract, the quantity discount contract, the incentive mechanism, etc. Zhao and Wang [34] made a profit improvement by extending the Stackelberg model with a proper contract design. Yu et al. [35] pointed out that the Stackelberg equilibrium can be improved to further benefit the producer and its retailers if the retailers are willing to cooperate with the producer by using a cooperative contract. In this paper, a particular revenue sharing contract is utilized by the players in the Stackelberg game for gaining a higher profit by channel coordination.

For the game theory with constraints, Breton and Zaccour [36] studied that the Stackelberg game with a security constraint for the follower and presented the equilibrium solution under the specific condition characterized by parameters. Shantha Daniel [37] solved that the Stackelberg model for the electricity firms in a successive structure with some constraints. Baumgartner and Jost [38] and Baumgartner [16] obtained the equilibrium solution by taking into account the joint production problems, where a constraint about the disposal of the excess of secondary resource is considered. Breton, etc. [39] studied several game theoretical models in environmental projects.

In this paper, mathematical programming models are built for two outputs (i.e., ethanol and DG) which necessarily emerge from a single activity of processing corn. So, the organization of for the Stackelberg model without constraint is described as follows. Chapter 2 sets up the model environment, including the definition and notation for the models, and states the assumption conditions; establishes the E&DG producer-driven Stackelberg model with the linear unit joint production cost; obtains the equilibrium solution in the ED model;

and makes the robust analysis to selected parameters in the Stackelberg model. In chapter 3, the centrally coordinated model (CC) on the vertical structure supply chain is extended and proposed as a benchmark so asfor a comparison with the Stackelberg model. In chapter 4, the E&DG producer-driven Stackelberg model with a quadratic unit joint production cost (EDQ) is developed, and then the comparison between the ED model and the EDQ model is executed. In chapter 5, the numerical applications for all models arepresent, with two numerical comparisons: 1) one is between the ED model and the CC model, and 2) one is between the ED model and the EDQ model. Chapter 6 concludes our findings and suggests future research.

2. THE E&DG PRODUCER-DRIVEN (ED) STACKELBERG MODEL

2.1 Model environment

The following model with an E&DG producer as well as a feed producer is inspired by the example of ethanol and DG from corn. Both ethanol and DG emerge from a single joint production activity of the E&DG producer. Ethanol is sold to various customers in the ethanol market at a certain price, such as BP, Conoco-Phillips, etc.. DG is sold to various feed producers with a certain price, e.g., Land O'Lakes, whom in turn, sells feed to customers in the feed market with a certain price, such as local feedlot. In this paper, we model this economical relationship between a single representative E&DG producer and a single representative feed producer as depicted in Figure 2.1. The E&DG producer is denoted as "he" and the feed producer is denoted as "she" where applicable throughout this paper.

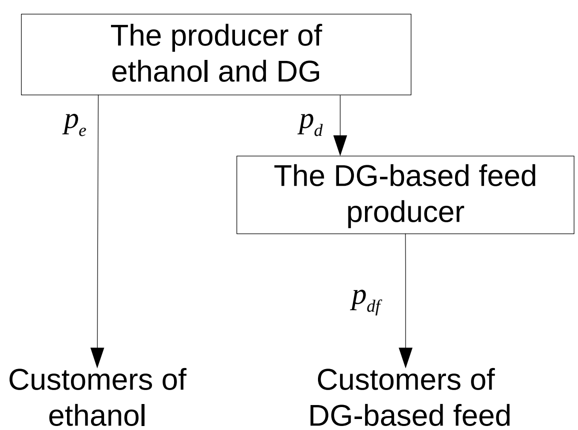


Figure 2.1 Configuration of the ED model with one E&DG producer and one feed producer

So far as we know, ethanol and DG are produced from corn in the production of the E&DG producer. Then, DG serves as input for the process of feed production. Feed consist of other ingredients and DG and is produced by the feed producer. In summary for transactions in the supply chain, the E&DG producer sells ethanol to customers of ethanol such as the ethanol-based fuel producer, and at the same time sells DG to the feed producer; the feed producer in turn sells feed to customers of DG-based feed in the feed market.

2.1.1 Definitions

On this subsection, we first present the following list of notations.

Parameters

k_e the proportion of ethanol produced from one unit of corn

k_d the proportion of DG produced from one unit of corn

c_{em} the joint production cost of processing one ton of corn (\$/ton)

c_c the corn price (\$/ton)

c_{fm} the processing cost to obtain one ton of feed (\$/ton)

c_f the cost of per ton of other ingredients in the feed production (\$/ton)

τ_f the fraction of DG in DG-based feed (the DG fraction)

p_e the price of ethanol (\$/ton)

α_{df} the maximum price of DG-based feed

β_{df} the price sensitivity to demand of feed

c_{dd} the drying cost of obtaining one ton of DG (\$/ton)

Decision variables for models

p_d the price of DG (\$/ton)

p_{df} the price of DG-based feed (\$/ton)

Dependent variables and other economic variables

D_{df} $D_{df} = (\alpha_{df} - p_{df}) / \beta_{df}$, the quantity of DG-based feed (ton)

D_d the demand quantity of DG (ton)

Q the quantity of corn used for the E&DG production (ton)

Q_d the quantity of DG produced from the E&DG production (ton)

Q_e the quantity of ethanol produced from the E&DG production (ton)

D_e the demand quantity of ethanol (ton)

$\prod_E$ the E&DG producer's profit (\$)

$\prod_F$ the feed producer's profit (\$)

$\prod_{CC}$ the total profit of the whole supply chain ($\$$)

R_e the revenue from selling ethanol ($\$$)

R_d the revenue from selling DG ($\$$)

R_{df} the revenue from selling DG-based feed ($\$$)

As for expenses, c_{em} is the cost to produce k_e tons of ethanol and k_d tons of DG from one ton of corn; c_c is the corn price per ton; c_{fm} is the processing cost when producing one ton of feed; and c_f is the cost of other ingredients (e.g., forage, alfalfa, corn, etc.). Thus, $c_f\left(1-\tau_f\right)$ is the cost of other ingredients in order for producing one ton of feed which contains τ_f tons of DG.

p_e is the price of ethanol sold by the E&DG producer to the ethanol market. p_d is the price of DG sold from the E&DG producer to the feed producer. p_{df} is the price of DG-based feed sold by the feed producer to customers in the feed market. In this paper, p_d, p_{df} are decision variables in the competition environment, and p_{df} is the decision variable in the coordinated environment.

For simplicity, we utilize "ton" to describe the measure unit of all relevant products (including corn, ethanol, DG, other ingredients, and feed), and use the conversion: 1 ton = 330 gallon to convert ethanol price, typically represented by $/gallon to $/ton; 1ton = 39.36 bushel to convert corn price, typically represented by $/bushel to $/ton [40].

2.1.2 Assumptions

A1. *The demand function of feed is assumed as* $D_{df}=\left(\alpha_{df}-p_{df}\right)/\beta_{df}$ *, which is widely utilized in some supply chain literature [27],[32],[33]. Thereby, the demand of feed* D_{df} *is a* *decreasing linear function with respect to* p_{df}. $\alpha_{df} > 0$ *is the maximum feed price which is greater than the optimal feed price* p_{df}, *and* $\beta_{df} > 0$ *is the price sensitivity to the demand of feed. We assume the quantity of ethanol, DG and feed is positive, the price of DG as well as feed is positive. The quantity of feed is* $D_{df} \in \left(0, \alpha_{df} / \beta_{df}\right)$, *and the feed price is* $p_{df} \in \left(0, \alpha_{df}\right)$.

The quantity of feed D_{df} is assumed to be positive, because the supply chain in Figure 2.1 does not exist if the quantity of feed is less than or equal to zero; the quantity of feed D_{df} is assumed to be less than α_{df} / β_{df}, because the DG price will be non-positive if D_{df} is greater than or equal to α_{df} / β_{df}. When the DG price is negative, the E&DG producer will discard DG freely in order to avoid the lose in selling DG, and when the DG price is zero, the E&DG producer will not like to cost in drying DG for nothing revenue from selling DG.

A2. k_e *and* k_d *are the fixed proportions, where* $k_e, k_d > 0$ *are constant values.*

Processing corn into ethanol and DG requires fixed proportions of ethanol (k_e) and DG (k_d) per unit of corn processed [7]. In addition, we assume DG is dried to 0% moisture, although most E&DG producers produced dry DG to about 10% moisture.

A3. *The customers in the ethanol market and the DG-based feed market respectively consume all produced ethanol and feed. In addition, all produced DG is sold to the feed producer as the raw material in the DG-based feed production and the feed producer purchases DG only from the E&DG producer.*

The quantity of DG sold to the feed producer is determined by the demand of feed and the DG fraction, $D_d = D_{df}\tau_f$. The E&DG producer exactly satisfies the demand of DG from the feed producer, $Q_d = D_d = D_{df}\tau_f$. The produced amount of ethanol is equal to the demand of ethanol from the ethanol market, $Q_e = D_e$. As shown in the assumption **A2**, k_e ton of ethanol and k_d ton of DG from are produced from one ton of corn. Given τ_f, producing one ton of feed requires τ_f ton of DG; τ_f ton of DG are produced from τ_f / k_d ton of corn; and τ_f / k_d ton of corn produce $k_e \tau_f / k_d$ ton of ethanol.

So, given D_{df}, the quantity of dried DG, the quantity of corn, and the quantity of ethanol are presented by the following:

1) Eq. (2.1), the quantity of dried DG produced by the E&DG producer is

$$Q_d = D_d = D_{df} \tau_f . \tag{2.1}$$

2) Eq. (2.2), the quantity of corn used for the production is

$$Q = Q_d / k_d = D_d / k_d = D_{df} \tau_f / k_d . \tag{2.2}$$

3) Eq. (2.3), the quantity of ethanol produced from the E&DG producer is

$$D_e = Q_e = k_e Q = k_e Q_d / k_d = k_e D_d / k_d = k_e D_{df} \tau_f / k_d . \tag{2.3}$$

A4. $0 < \tau_f < 1$, *the DG fraction is greater than zero and less than one. The customers in the feed market make no distinction to feed with the different DG fraction.*

It is the nonexistence of the supply chain as shwon in Figure 2.1 since the feed producer does not purchase any DG while $\tau_f = 0$. And it is unrealistic to feed animals with 100% DG, $\tau_f = 1$ [11]. The customer in the feed market makes no distinction to feed with different DG fraction since feed is assumed to have the same quality with different DG fraction.

A5. *The E&DG producer is considered as a price-taker on ethanol as mentioned in the introduction [7],[11]. However, the E&DG producer is the price-maker on DG in this two-player vertical supply chain structure.*

The price the E&DG producer sets and the quantity of ethanol this producer produces do not have any effect on the market price. Therefore, the E&DG producer does not need to worry about what price to set for ethanol. Instead, E&DG producer is concerned with only how much to produce [16], since an E&DG producer is small when compared to the ethanol market, i.e., all ethanol producers in American have no dominant market power in the ethanol market [8]. However, the E&DG producer can control the selling price of DG to the feed produce, since most feed producers are small when comparing to the E&DG producer and set up their facility around the E&DG producer for avoiding more cost in transportation.

A6. *There are no fixed costs for the ethanol and DG production and the DG-based feed production respectively for both producers, since the issue related to investment is not explored.*

A7. *Each producer has positive profit. And, it is a static single period strategy for producers.*

Here we use Eq. (2.4) to denote the gain of the feed producer from one ton of feed and Eq.(2.5) to denote the gain of the E&DG producer from one ton of DG, where both unit profits are positive since both producers need get benefit from this supply chain.

$$p_{df} - c_f\left(1-\tau_f\right) - c_{fm} - p_d\tau_f \qquad (2.4)$$

$$\left(p_e k_e + p_d k_d - \left(c_{em} + c_c + c_{dd}k_d\right)\right)/k_d \qquad (2.5)$$

1) The E&DG producer

We re-arrange the gain of the E&DG producer from one ton of DG to one ton of corn,

$$p_d k_d - \left(\left(c_{em} + c_c + c_{dd}k_d\right) - p_e k_e\right). \qquad (2.6)$$

From the assumption **A5**, the E&DG producer is the price taker on ethanol and the price maker on DG. He has the difference in Eq. (2.7) between the sum of all costs for processing one ton of corn (including the joint production cost, the corn price, and drying cost for DG) and the revenue from selling the ethanol produced from one ton of corn, $\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e$, that is the benefit from one ton of corn excluding the revenue from DG,

$$\mathrm{A}=\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e . \tag{2.7}$$

a) $\mathrm{A}\leq 0$, the revenue from selling ethanol totally covers all cost in producing ethanol and DG from one ton of corn. Even though the DG price is small, the E&DG producer always has profit through selling ethanol in the ethanol market.

b) $\mathrm{A}>0$, the revenue from selling ethanol is less than all cost in producing ethanol and DG from one ton of corn. The E&DG producer is small enough so that he can't cover all costs by selling ethanol and should have enough revenue through selling DG so as to cover the remaining cost.

2) The feed producer

The gain of the feed producer from one ton of feed in Eq. (2.4) is positive, that is, $p_{df}-c_f\left(1-\tau_f\right)-c_{fm}-p_d\tau_f>0$. According to the assumption **A1**, DG price is positive. Hence, from the assumption **A7**, the feed producer has the positive difference in Eq. (2.8) to show the benefit expressed in term of one ton of feed excluding the cost from DG,

$$0<p_{df}-c_f\left(1-\tau_f\right)-c_{fm} . \tag{2.8}$$

Moreover, since α_{df} is the maximum feed price, the term $\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}$ that is the difference between the maximum feed price and the sum of the processing cost and the cost from other ingredients for producing one ton of feed should be satisfied by Eq. (2.9).

$$0 < \alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm} \tag{2.9}$$

where $c_f\left(1-\tau_f\right)$ is the other ingredients' cost for one ton of feed, c_{fm} is the processing cost for one ton of feed and α_{df} is the maximum price per ton of feed.

For any $\tau_f, \tau_f \in (0,1)$, there is Eq. (2.10) derived from for the assumptions **A1** and **A7**,

$$0 < \alpha_{df} - c_f - c_{fm}. \tag{2.10}$$

Based on the configuration and assumptions, the next section will study the equilibrium in the Stackelberg model, where the E&DG producer has leadership over the feed producer.

2.1.3 Scope

1) In this configuration, there are no other usages of corn except for as the raw material in the corn-ethanol&DG production and the possible component of other ingredients in the feed production.
2) Corn and DG are not sold directly to the cumstomers in the feed market. So, there is no competitive substitution relationship among corn, DG, and DG-based feed in the feed market to feed animal. And there is no competitive substitution relationship between corn for the corn-ethanol&DG production and corn as the component of other ingredients for the feed production.
3) Ethanol in the ethanol market and DG for the feed production is produced only from corn. Other ingredients (e.g., forage, alfalfa, corn, etc.) can be assumed as one raw material for the feed production.

2.2 The E&DG Producer-Driven Stackelberg model (ED)

In this section, a detailed formulation and analysis of a Stackelberg model are presented for the E&DG producer and the feed producer, where the former is the Stackelberg leader and the latter is the Stackelberg follower. The Stackelberg model is a strategic game where the leader moves first and the follower moves sequentially, after observing the leader's decision. In turn, the leader knows the follower will make her own decision, after observing the leader's decision [41]. This model is designated as "ED."

As for the feed transaction, the feed producer charges the price p_{df} per unit of feed and receives payment $p_{df}D_{df}$ from customers in the feed market. For the DG transaction, the E&DG producer only charges p_d per unit of DG and receives payment $p_d D_{df} \tau_f$ from the feed producer according to Eq.(2.1). For the ethanol transaction, the E&DG producer charges p_e per unit of ethanol and receives payment $p_e D_e = p_e k_e D_{df} \tau_f / k_d$ from customers in the ethanol market according to Eq.(2.3).

2.2.1 Profit maximum problems and the equilibrium solution

In the ED model, the equilibrium solution is characterized via backward induction by first characterizing the response function of the feed producer. Then, this is followed by the solution to the E&DG producer's profit problem.

1) The feed producer

From the assumption **A3** the feed producer only purchases DG from the E&DG producer and then sells all DG-based feed to customers in the feed market. The profit problem of the feed producer is shown in Eq.(2.11),

$$\Pi_F = R_{df} - p_d D_d - c_f\left(D_{df} - D_d\right) - c_{fm} D_{df} \quad (2.11)$$

The profit for the feed producer, $\prod_F$, is given by the difference between the revenue of selling feed and all relevant costs. The first term R_{df} is the revenue from selling feed and equals to $p_{df} D_{df}$; the second term $p_d D_d$ is the cost of purchasing DG from the E&DG producer; the third term $c_f \left(D_{df} - D_d \right)$ is the cost of purchasing other ingredients; and the last term $c_{fm} D_{df}$ is the processing cost for the feed production.

From the assumption **A1** and Eqs. (2.1),(2.2),(2.3) and (2.11), the feed producer maximized her profit function in Eq.(2.12), where she decides the price of feed p_{df} and assumes the price of DG p_d given.

$$\begin{aligned} \underset{p_{df}}{\text{Max}} \quad \prod_F &= p_{df} D_{df} - \bar{p}_d D_{df} \tau_f - c_f \left(1 - \tau_f\right) D_{df} - c_{fm} D_{df} \\ &= \left(p_{df} - \bar{p}_d \tau_f - c_f \left(1 - \tau_f\right) - c_{fm} \right) \frac{\alpha_{df} - p_{df}}{\beta_{df}} \end{aligned} \qquad (2.12)$$

Given p_d , the concavity of the profit function of the feed producer $\prod_F$ in the price of feed p_{df} is guaranteed by the second-order sufficient condition,

$$\partial^2 \prod_F / \partial p_{df}{}^2 = -2 / \beta_{df} < 0 . \qquad (2.13)$$

The first-order necessary condition of Eq. (2.12) with respect to the price of feed p_{df} is the equilibrium condition of the feed producer, as shown in Eq. (2.14),

$$\frac{\partial \prod_F}{\partial p_{df}} = \frac{\alpha_{df} + c_f \left(1 - \tau_f\right) + c_{fm} + \tau_f \bar{p}_d - 2 p_{df}}{\beta_{df}} = 0 . \qquad (2.14)$$

Moreover, Eq. (2.14) is rearranged to the explicit form of the unique best response function of the feed producer for the given DG price p_d , shown as follows,

$$\hat{p}_{df}(p_d): \qquad \hat{p}_{df}(\overline{p}_d) = \frac{\alpha_{df} + c_f\left(1-\tau_f\right) + c_{fm}}{2} + \frac{\tau_f}{2}\,\overline{p}_d \quad (2.15)$$

So far, the E&DG producer can decide the optimal price for DG p_d after knowing about the best response function of the feed producer $\hat{p}_{df}(p_d)$ in Eq. (2.15).

2) The E&DG producer

As for the E&DG producer, he sells DG to the feed producer and ethanol to customers in the ethanol market, and generates the costs in corn, the joint production, and the drying process of DG. Then, the E&DG producer's profit problem is shown in Eq. (2.16),

$$\prod_E = R_d + R_e - c_{em}Q - c_cQ - c_{dd}D_d \quad (2.16)$$

The profit of the E&DG producer, $\prod_E$, is given by the difference between the revenues of selling DG and ethanol, on the one hand, and the cost of the E&DG production, on the other hand. The first term R_d is the revenue from selling DG and equals to p_dD_d; the second term R_e is the sum of the revenue from selling ethanol and equals to p_eD_e; the third term $c_{em}Q$ is the joint production cost for the E&DG producer to process Q tons of corn; the fourth term c_cQ is the cost of corn; and the final term $c_{dd}D_d$ is the drying cost of obtaining $D_d = k_dQ$ tons of DG.

From the assumption **A1** and Eqs. (2.1), (2.2), (2.3)and (2.16), the E&DG producer maximizes his own profit function in Eq. (2.17), where he decides the price of DG p_d, and has known the best response function $\hat{p}_{df}(p_d)$ which is the function of p_d in Eq. (2.15).

$$\underset{p_d}{\text{Max}} \quad \Pi_E = p_e \frac{k_e D_{df} \tau_f}{k_d} + p_d D_{df} \tau_f - c_{em} \frac{D_{df} \tau_f}{k_d} - c_c \frac{D_{df} \tau_f}{k_d} - c_{dd} D_{df} \tau_f$$
$$= \left(p_e \frac{k_e}{k_d} + p_d - \frac{(c_{em} + c_c + c_{dd} k_d)}{k_d} \right) \frac{\alpha_{df} - \hat{p}_{df}(p_d)}{\beta_{df}} \tau_f \qquad (2.17)$$

After substituting the best response function $\hat{p}_{df}(p_d)$ in Eq. (2.15) into Eq. (2.17), the concavity of the profit function of the E&DG producer in the price of DG p_d is guaranteed by the second-order sufficient condition in Eq. (2.18):

$$\partial^2 \Pi_E / \partial p_d^{\ 2} = -\tau_f^{\ 2} / \beta_{df} < 0 \qquad (2.18)$$

Hence, Eq. (2.19), the first-order necessary condition of Eq. (2.17) in p_d , is the equilibrium condition of the E&DG producer,

$$\frac{\partial \Pi_E}{\partial p_d} = \frac{\left(\left(\alpha_{df} - c_f \left(1 - \tau_f\right) - c_{fm} \right) k_d + \mathrm{A} \tau_f \right) \tau_f - 2 k_d \tau_f^{\ 2} p_d}{2 \beta_{df} k_d} = 0 \qquad (2.19)$$

where $\mathrm{A} = \left(c_{em} + c_c + c_{dd} k_d\right) - k_e p_e$. In Eq. (2.19) , the term $\alpha_{df} - c_f\left(1 - \tau_f\right) - c_{fm}$ should be nonnegative since from the assumptions **A7** and **A1**. In summary, the equilibrium conditions consist of Eqs. (2.14) and (2.19). Therefore, after solving the equilibrium conditions, the equilibrium DG price in the ED model is shown in Eq. (2.20), where the superscript *ed* denotes the ED model and * is the designation of optimality.

$$p_d^{\ ed*} = \frac{\left(\alpha_{df} - c_f \left(1 - \tau_f\right) - c_{fm} \right) k_d + \mathrm{A} \tau_f}{2 k_d \tau_f} \qquad (2.20)$$

The DG price $p_d^{\ ed*}$ in Eq. (2.20) is assumed as positive since otherwise the E&DG producer would have nothing revenue from selling DG with a negative DG price from the assumption **A1**. Thus,

$$-\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d < \mathrm{A}\tau_f \, . \tag{2.21}$$

After substituting the equilibrium DG price into the best response function in Eq. (2.15), the equilibrium feed price is,

$$p_{df}^{ed*} = \frac{\left(3\alpha_{df} + c_f\left(1-\tau_f\right) + c_{fm}\right)k_d + \mathrm{A}\tau_f}{4k_d} \, . \tag{2.22}$$

As we know from the assumption **A1**, the feed price p_{df}^{ed*} should be positive. Eq. (2.23) is positive due to $-\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d < \mathrm{A}\tau_f$ in Eq. (2.21). Then the feed price in Eq. (2.22) should be positive accoridng to Eq. (2.21).

$$\left(3\alpha_{df} + c_f\left(1-\tau_f\right) + c_{fm}\right)k_d + \mathrm{A}\tau_f > 2\alpha_{df} > 0 \tag{2.23}$$

where α_{df} is positive.

Table 2.1 shows the equilibrium solution corresponding to the quantities and the profits in the supply chain.

Table 2.1 The equilibrium solution of the ED model

D_{df}^{ed*}	$\frac{\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f}{4k_d\beta_{df}}$
D_d^{ed*}	$\frac{\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f}{4k_d\beta_{df}}\tau_f$
Q^{ed*}	$\frac{\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f}{4k_d^2\beta_{df}}\tau_f$

D_e^{ed*}	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{4k_d^{\,2}\beta_{df}}k_e\tau_f$
$p_{df}^{\;ed*}$	$\frac{\left(4\alpha_{df}-\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)\right)k_d+\mathrm{A}\tau_f}{4k_d}$
$p_d^{\;ed*}$	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d+\mathrm{A}\tau_f}{2k_d\tau_f}$
$\Pi_E^{\;ed*}$	$\frac{\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)^2}{8k_d^{\,2}\beta_{df}}$
$\Pi_F^{\;ed*}$	$\frac{\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)^2}{16k_d^{\,2}\beta_{df}}$

where $\mathrm{A}=\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e$ as in Eq. (2.7).

$$D_{df}^{\;ed*}=\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{4k_d\beta_{df}}. \tag{2.24}$$

$D_{df}^{\;ed*}$ in Eq. (2.24) is ranged in $D_{df}\in\left(0,\alpha_{df}/\beta_{df}\right)$ from the assumption **A1**. Therefore, the optimal feed quantity is greater than zero.

$$0<D_{df}^{\;ed*}=\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{4k_d\beta_{df}}. \tag{2.25}$$

Then, Eq. (2.25) is rearranged as Eq. (2.26),

$$\mathrm{A}\tau_f<\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d. \tag{2.26}$$

In addition, the optimal feed quantity is less than α_{df}/β_{df} from the assumption **A1**. That is,

$$D_{df}^{ed*} = \frac{\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f}{4k_d\beta_{df}} < \alpha_{df} / \beta_{df} \tag{2.27}$$

Moreover, Eq. (2.27) is rearranged

$$-\left(3\alpha_{df} + c_f\left(1-\tau_f\right) + c_{fm}\right)k_d < \mathrm{A}\tau_f \,. \tag{2.28}$$

Eq. (2.27) is satisfied according to Eq. (2.23) if $-\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d < \mathrm{A}\tau_f$ in Eq. (2.21) is true.

In sum, it should be observed from Eq. (2.21) for and Eq. (2.26) that the following condition in Eq. (2.29) for the positive DG price p_d^{ed*} and the positive quantity of feed D_{df}^{ed*} from the assumption **A1**.

$$-\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d < \mathrm{A}\tau_f < \left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d \,, \tag{2.29}$$

where $\mathrm{A} = \left(c_{em} + c_c + c_{dd}k_d\right) - k_e p_e$ as in Eq. (2.7).

Eq. (2.29) is rearranged to be Eq. (2.30) for the positive DG fraction τ_f ,

$$\frac{-\left(\alpha_{df} - c_f - c_{fm}\right)k_d}{\tau_f} - c_f k_d < \mathrm{A} < \frac{\left(\alpha_{df} - c_f - c_{fm}\right)k_d}{\tau_f} + c_f k_d \,. \tag{2.30}$$

Eq. (2.30) is hold under the condition Eq. (2.31) for any $\tau_f, \tau_f \in (0,1)$ in the assumption **A4**,

$$-\left(\alpha_{df} - c_{fm}\right)k_d < \mathrm{A} < \left(\alpha_{df} - c_{fm}\right)k_d \,, \tag{2.31}$$

where $\alpha_{df} - c_{fm} > 0$ according to the assumptions **A1** and**A7**.

The proof for Eq. (2.31) is shown in the following:

In Eq. (2.30), for any $\tau_f, \tau_f \in (0,1)$,

a) the term $\left(\alpha_{df} - c_f - c_{fm}\right)k_d / \tau_f + c_f k_d$ has the minimum value $\left(\alpha_{df} - c_f - c_{fm}\right)k_d + c_f k_d$ when $\tau_f \to 1$, thus $\mathrm{A} < \left(\alpha_{df} - c_f - c_{fm}\right)k_d + c_f k_d$, that is, $\mathrm{A} < \left(\alpha_{df} - c_{fm}\right)k_d$;

b) the term $-\left(\alpha_{df} - c_f - c_{fm}\right)k_d / \tau_f - c_f k_d$ has the maximum value $-\left(\alpha_{df} - c_f - c_{fm}\right)k_d - c_f k_d$ when $\tau_f \to 1$, thus $-\left(\alpha_{df} - c_f - c_{fm}\right)k_d - c_f k_d < \mathrm{A}$, that is, $-\left(\alpha_{df} - c_{fm}\right)k_d < \mathrm{A}$.

So, for and any $\tau_f, \tau_f \in (0,1)$ Eq. (2.31) should be satisfied. Q.E.D.

From Table 2.1, we note that

$$\prod_E{}^{ed*} = 2\prod_F{}^{ed*}. \tag{2.32}$$

That is, the profit of the E&DG producer is twice as much as that of the feed producer at the equilibrium point, since the E&DG producer has the first-move advantage as the leadership to know the best response function from the feed producer and set up a DG price which can be accepted by the feed producer in the supply chain.

Eq. (2.32) is satisfied, under the conditions as below,

1) The assumption **A3** shows that $k_d Q = D_d$, all DG produced by the E&DG producer is consumed by the feed producer for the animal feed production;
2) the E&DG producer is using a linear pricing policy for DG;
3) and the feed producer is facing a linear demand function of feed;
4) both producers have the linear production cost.

The feed producer has the best response in feed price in Eq. (2.15) with regard to the DG price. Therefore, for any p_d given by the E&DG producer, there exists the corresponding feed price. Then, substituting Eq. (2.15) into the profit problem of the feed producer in Eq. (2.12) as well as the E&DG producer in Eq. (2.17), we have $\prod_E{}^{ed*} = 2\prod_F{}^{ed*}$ at the equilibrium point $p_d{}^{ed*} = \left(\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d + \mathrm{A}\tau_f\right)/\left(2k_d\tau_f\right)$.

At the equilibrium point, the feed producer's gain in terms of feed is shown in Eq. (2.33).

$$
\begin{aligned}
& p_{df}{}^{ed*} - c_f\left(1-\tau_f\right) - c_{fm} - p_d{}^{ed*} \\
& = \left(\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f\right)/\left(4k_d\right)
\end{aligned}
\tag{2.33}
$$

According to Eq. (1), $D_d = D_{df}\tau_f$. Hence, from Eq. (2.5), the E&DG producer's gain in terms of feed is shown in Eq. (2.34).

$$
\begin{aligned}
& \left(p_d{}^{ed*} - \left(c_{em} + c_c + c_{dd}k_d\right)/k_d + p_e k_e / k_d\right)\tau_f \\
& = \left(\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right) - \mathrm{A}\tau_f\right)/\left(2k_d\right)
\end{aligned}
\tag{2.34}
$$

From Eqs. (2.33) and (2.34), the E&DG producer obtains twice as much profit from one ton of DG as the feed producer.

2.3 Analysis of the other ingredients' cost c_f and the DG fraction τ_f

In the U.S.A., the optimal DG fraction often is studied for the animal feeding performance. Also, the feed producer is coming up against the problem that sales would not cover higher production costs [42],[43],[44], because of the higher other ingredients' cost (i.e., corn, alfalfa). However, the increase of the DG fraction reduces the higher production cost to replace part of corn in feed when the DG price (corn basis) is lower than the corn price.

Based on this reason, the impact from the other ingredients' cost c_f and the DG fraction τ_f on the supply chain is worth to be explored.

2.3.1 Effects of the cost of other ingredients c_f

Derivative values of p_d^{ed*}, p_{df}^{ed*}, D_d^{ed*}, D_{df}^{ed*}, $\prod_E^{ed*}$ and $\prod_F^{ed*}$ with respect to c_f, respectively are studied and shown in Table 2.2.

Table 2.2 The derivative analysis of the equilibrium solution

	c_f
p_d^{ed*}	$\frac{dp_d^{ed*}}{dc_f}=-\frac{1-\tau_f}{2\tau_f}$
p_{df}^{ed*}	$\frac{dp_{df}^{ed*}}{dc_f}=\frac{1-\tau_f}{4}$
D_d^{ed*}	$\frac{dD_d^{ed*}}{dc_f}=-\frac{1-\tau_f}{4\beta_{df}k_d}\tau_f$
D_{df}^{ed*}	$\frac{dD_{df}^{ed*}}{dc_f}=-\frac{1-\tau_f}{4\beta_{df}}$
$\prod_E^{ed*}$	$\frac{d\prod_E^{ed*}}{dc_f}=\frac{-(1-\tau_f)\left(\left(\alpha_{df}-c_f(1-\tau_f)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)}{4\beta_{df}k_d}$
$\prod_F^{ed*}$	$\frac{d\prod_F^{ed*}}{dc_f}=\frac{-(1-\tau_f)\left(\left(\alpha_{df}-c_f(1-\tau_f)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)}{8\beta_{df}k_d}$

where $\mathrm{A}=(c_{em}+c_c+c_{dd}k_d)-k_e p_e$ as in Eq. (2.7).

Table 2.2 shows that, as the cost of other ingredients c_f increases, the DG price and the quantity of DG decrease, the feed price increases and the quantity of feed decreases. As c_f

increases, the feed price increases in contrast with the price of DG decreases according to Table 2.2. The increase of p_{df}^{ed*} will result in a decrease of the quantity of feed because of the down sloping demand function in the assumption **A1**.

$$\frac{dp_{df}^{ed*}}{dc_f} = -\frac{\tau_f}{2}\frac{dp_d^{ed*}}{dc_f} \tag{2.35}$$

$$\frac{dD_{df}^{ed*}}{dc_f} = \frac{k_d}{\tau_f}\frac{dD_d^{ed*}}{dc_f} \tag{2.36}$$

Eq. (2.35) shows that the feed price is less sensitive to change in c_f than the price of DG, due to $\tau_f < 1$. Eq. (2.36) shows that the quantity of feed as well as DG decreases as c_f increases.

PROPOSITION 1. If the cost of one ton of other ingredients c_f changes, then the profit of the E&DG producer changes twice higher than that of the feed producer in the ED model.

Proof: In Table 2.2, there exist

$$\frac{d\prod_E^{ed*}}{dc_f} = \frac{-\left(1-\tau_f\right)\left(\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f\right)}{4\beta_{df}k_d} \tag{2.37}$$

$$\frac{d\prod_F^{ed*}}{dc_f} = \frac{-\left(1-\tau_f\right)\left(\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f\right)}{8\beta_{df}k_d} \tag{2.38}$$

where $\mathrm{A} = \left(c_{em} + c_c + c_{dd}k_d\right) - k_e p_e$ as in Eq. (2.7). In addition, the term in Eq. (2.37) and (2.38), $\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f > 0$ due to the assumption **A1** that the quantity of feed is positive.

Thus, for any $\tau_f, 0 < \tau_f < 1$, Eq. (2.10) is rearranged to be Eq. (2.39) where the other ingredients' cost c_f is less than $\alpha_{df} - c_{fm}$,

$$c_f < \alpha_{df} - c_{fm} \tag{2.39}$$

As c_f increases, $\prod_E^{ed*}$ and $\prod_F^{ed*}$ will decrease. And from Eq. (2.40), it can be observed that the E&DG producer will have the profit loss twice higher than the feed producer as c_f increases, because of $\prod_E^{ed*} = 2\prod_F^{ed*}$ from Table 2.1 for any given c_f. Q.E.D.

$$\left|\frac{d\prod_E^{ed*}}{dc_f}\right| = 2\left|\frac{d\prod_F^{ed*}}{dc_f}\right| \tag{2.40}$$

Therefore, the profit of the E&DG producer is more sensitive to changes in the other ingredients' cost, c_f, than that of the feed producer.

2.3.2 Effects of the DG fraction τ_f

From two reasons: there is the assumption **A4** that $0 < \tau_f < 1$, and the increase of the DG fraction reduces the higher production cost to replace part of other ingredients in feed when the DG price (corn basis) is lower than the cost of other ingredients, here we would like to discuss the impact from the DG fraction τ_f in the range of $(0,1)$. Derivative values of p_d^{ed*}, $p_{df}^{ed*}, D_d^{ed*}, D_{df}^{ed*}, \prod_E^{ed*}$ and $\prod_F^{ed*}$ with respect to τ_f are studied and shown in Table 2.3.

Table 2.3 The derivative analysis of the equilibrium solution w.r.t the DG fraction

	τ_f
p_d^{ed*}	$\frac{dp_d^{ed*}}{d\tau_f} = -\frac{\alpha_{df} - c_f - c_{fm}}{2\tau_f^2}$

p_{df}^{ed*}	$\frac{dp_{df}^{ed*}}{d\tau_f}=\frac{A-c_f k_d}{4k_d}$
D_d^{ed*}	$\frac{dD_d^{ed*}}{d\tau_f}=\frac{(\alpha_{df}-c_f-c_{fm})k_d-2(A-c_f k_d)\tau_f}{4\beta_{df}k_d}$
D_{df}^{ed*}	$\frac{dD_{df}^{ed*}}{d\tau_f}=-\frac{A-c_f k_d}{4k_d\beta_{df}}$
$\prod_E^{ed*}$	$\frac{d\prod_E^{ed*}}{d\tau_f}=\frac{-(A-c_f k_d)((\alpha_{df}-c_f(1-\tau_f)-c_{fm})k_d-A\tau_f)}{4\beta_{df}k_d^2}$
$\prod_F^{ed*}$	$\frac{d\prod_F^{ed*}}{d\tau_f}=\frac{-(A-c_f k_d)((\alpha_{df}-c_f(1-\tau_f)-c_{fm})k_d-A\tau_f)}{8\beta_{df}k_d^2}$

where $A=(c_{em}+c_c+c_{dd}k_d)-k_e p_e$ as in Eq. (2.7).

<u>PROPOSITION 2</u>. If the DG fraction τ_f changes, then the profit of the E&DG producer changes twice higher than that of the feed producer in the ED model.

Proof: In Table 2.3, there exist

$$\frac{d\prod_E^{ed*}}{d\tau_f}=\frac{-(A-c_f k_d)((\alpha_{df}-c_f(1-\tau_f)-c_{fm})k_d-A\tau_f)}{4\beta_{df}k_d^2}, \qquad (2.41)$$

$$\frac{d\prod_F^{ed*}}{d\tau_f}=\frac{-(A-c_f k_d)((\alpha_{df}-c_f(1-\tau_f)-c_{fm})k_d-A\tau_f)}{8\beta_{df}k_d^2}. \qquad (2.42)$$

where $A=(c_{em}+c_c+c_{dd}k_d)-k_e p_e$ as in Eq. (7). In addition, the term $(\alpha_{df}-c_f(1-\tau_f)-c_{fm})k_d-A\tau_f>0$ due to the assumption **A1** that the quantity of feed is positive.

So, from Eqs. (2.41) and (2.42) we can find

$$\left|d\prod_E{}^{ed*}/d\tau_f\right| = 2\left|d\prod_F{}^{ed*}/d\tau_f\right|. \tag{2.43}$$

From Eq. (2.43), the profit of the E&DG producer is more sensitive to changes in the DG fraction than that of the feed producer. Q.E.D.

There exist the conditions from Eq. (2.10) with $0 < \alpha_{df} - c_f - c_{fm}$ and Eq. (2.31) with $-\left(\alpha_{df} - c_{fm}\right)k_d < \mathrm{A} < \left(\alpha_{df} - c_{fm}\right)k_d$, when the DG fraction τ_f is ranged in 0 and 1 as shown in the assumption **A4**.

$$0 < \tau_f < 1. \tag{2.44}$$

For any positive k_d, three cases derived from Table 2.3 are listed as below:

Case 1: $\mathrm{A} > c_f k_d$. That is $c_f < \mathrm{A}/k_d$, (2.45)

Case 2: $\mathrm{A} = c_f k_d$. That is $c_f = \mathrm{A}/k_d$, (2.46)

Case 3: $\mathrm{A} < c_f k_d$. That is $c_f > \mathrm{A}/k_d$, (2.47)

where $\mathrm{A} = \left(c_{em} + c_c + c_{dd}k_d\right) - k_e p_e$ as in Eq. (2.7). In addition, these three cases are three ones under different level of the other ingredients' cost.

From Table 2.1, the equilibrium DG price is re-arranged as (2.48),

$$p_d{}^{ed*} = \frac{\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d + \mathrm{A}\tau_f}{2k_d\tau_f} = \frac{\left(\alpha_{df} - c_f - c_{fm}\right)}{2\tau_f} + \frac{\mathrm{A} + c_f k_d}{2k_d}. \tag{2.48}$$

Thus, in Eq. (2.48) the DG price $p_d{}^{ed*}$ decreases as the DG fraction τ_f increases, according to condition in Eq. (2.10) $0 < \alpha_{df} - c_f - c_{fm}$.

1) Case 1: $\mathrm{A} > c_f k_d$, where $\mathrm{A} = \left(c_{em} + c_c + c_{dd}k_d\right) - k_e p_e$.

That is $c_f < A / k_d$ from Eq. (2.45). We re-write the expressions of parameters in Case 1 as following,

$$0 < A - p_d k_d + \left(p_d - c_f \right) k_d . \tag{2.49}$$

Note that $A - p_d k_d = \left(c_{em} + c_c + c_{dd} k_d \right) - k_e p_e - p_d k_d$ should be negative since $-\left(\left(c_{em} + c_c + c_{dd} k_d \right) - k_e p_e - p_d k_d \right)$ is equal to the profit from one ton of processed corn.

To guarantee that the E&DG producer has the positive profit by the term $0 < -\left(A - p_d k_d \right)$, the DG price is greater than the price of other ingredients,

$$c_f < -\left(A - p_d k_d \right) / k_d + c_f < p_d . \tag{2.50}$$

In Case 1: $A > c_f k_d$ with conditions in Eqs. (2.10) and (2.31), as τ_f increases, p_d^{ed*}, D_{df}^{ed*}, $\prod_F^{ed*}$ and $\prod_E^{ed*}$ keep decreasing, p_{df}^{ed*} increases and D_d^{ed*} decreases when $\tau_f \geq \left(\alpha_{df} - c_f - c_{fm} \right) k_d / \left(2A - 2c_f k_d \right)$ and increases when $\tau_f < \left(\alpha_{df} - c_f - c_{fm} \right) k_d / \left(2A - 2c_f k_d \right)$.

From Table 2.1, when $\tau_f \to 0$, both producers have the maximum profit, however, the quantity of DG as well as ethanol is close to zero, which is running in the opposite direction with the expanding ethanol market.

2) Case 2: $A = c_f k_d$, where $A = \left(c_{em} + c_c + c_{dd} k_d \right) - k_e p_e$.

That is $c_f = A / k_d$ from Eq. (2.46). We re-write the expressions of parameters in Case 2 as following,

$$A - p_d k_d + \left(p_d - c_f \right) k_d = 0 , \tag{2.51}$$

Similarly, to guarantee that the E&DG producer has the positive profit by the term $-\left(\mathrm{A}-p_d k_d\right)>0$, the DG price is greater than the price of other ingredients,

$$c_f < -\left(\mathrm{A}-p_d k_d\right)/k_d + c_f = p_d . \tag{2.52}$$

In Case 2: $\mathrm{A}=c_f k_d$ with conditions in Eqs. (2.10) and (2.31), as τ_f increases, $p_d^{\;ed*}$ keeps decreasing; $p_{df}^{\;ed*}$, $D_{df}^{\;ed*}$, $\prod_F^{\;ed*}$ and $\prod_E^{\;ed*}$ do not change; however, the quantity of DG $D_d^{\;ed*}$ increases since $dD_d^{\;ed*}/d\tau_f$ in Table 2.3 is positive.

$$\frac{dD_d^{\;ed*}}{d\tau_f} = \frac{\left(\alpha_{df}-c_f-c_{fm}\right)k_d - 2\left(\mathrm{A}-c_f k_d\right)\tau_f}{4\beta_{df} k_d} = \frac{\left(\alpha_{df}-c_f-c_{fm}\right)}{4\beta_{df}} > 0 , \tag{2.53}$$

if the condition $\alpha_{df}-c_f-c_{fm}>0$ in Eq. (2.10) is true. In addition, the quantity of ethanol increases according to Eq. (2.3), which is helpful to the expanding ethanol market.

3) Case 3: $\mathrm{A}<c_f k_d$, where $\mathrm{A}=\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e$.

That is $c_f > \mathrm{A}/k_d$ from Eq. (2.47). We re-write the expressions of parameters in Case 3 as following,

$$\mathrm{A}-p_d k_d + \left(p_d - c_f\right)k_d < 0 , \tag{2.54}$$

Similarly, to guarantee above equations with the positive value of $-\left(\mathrm{A}-p_d k_d\right)$, the DG price can be greater than, equal to, or less than the price of other ingredients.

In Case 3: $\mathrm{A}<c_f k_d$ with conditions in Eqs. (2.10) and (2.31), as τ_f increases, $p_d^{\;ed*}$ and $p_{df}^{\;ed*}$ keep decreasing, however, $D_d^{\;ed*}$, $D_{df}^{\;ed*}$, $\prod_F^{\;ed*}$ and $\prod_E^{\;ed*}$ increases. The quantity of DG $D_d^{\;ed*}$ increases since $dD_d^{\;ed*}/d\tau_f$ in Table 2.3 is positive.

$$\frac{dD_d^{ed*}}{d\tau_f}=\frac{\left(\alpha_{df}-c_f-c_{fm}\right)k_d-2\left(\mathrm{A}-c_f k_d\right)\tau_f}{4\beta_{df}k_d}>\frac{\left(\alpha_{df}-c_f-c_{fm}\right)}{4\beta_{df}}>0\,. \tag{2.55}$$

Moreover, the quantity of ethanol increases from Eq. (2.3) that there is the fixed proportion in the quantity between ethanol and DG, which is helpful to the expanding ethanol market.

In sum, the analysis for these 3 Cases is summarized in Table 2.4. For Case 1 and Case 2, the DG price should be greater than other ingredients' cost in order to guarantee the positive profit for the E&DG producer; for Case 3, however, there is no such condition relative to the DG price and other ingredients' cost.

Table 2.4 Condition analysis for each case

Case	Conditions	Derived conditions when $0<\tau_f<1$	The DG price	τ_f
1	$c_f<\mathrm{A}/k_d$	$0<\alpha_{df}-c_f-c_{fm}$, $-\left(\alpha_{df}-c_{fm}\right)k_d<\mathrm{A}<\left(\alpha_{df}-c_{fm}\right)k_d$	$p_d>c_f$	$0<\tau_f<1$
2	$c_f=\mathrm{A}/k_d$		$p_d>c_f$	$0<\tau_f<1$
3	$c_f>\mathrm{A}/k_d$		$p_d>c_f$, $p_d=c_f$, $p_d<c_f$	$0<\tau_f<1$

where $\mathrm{A}=\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e$ as shown in Eq. (2.7).

Under different cases, we summarize the change of p_d^{ed*}, p_{df}^{ed*}, D_d^{ed*}, D_{df}^{ed*}, $\prod_F^{ed*}$ and $\prod_E^{ed*}$ in Table 2.5, as the DG fraction τ_f increases. In Table 2.5, $\downarrow$, $=$, $\uparrow$ represent decreasing, no change, and increasing, respectively.

Table 2.5 The change of equilibrium solution as the DG fraction increases

	Case 1	Case 2	Case 3

$A - c_f k_d$	> 0	$= 0$	< 0
p_d^{ed*}	$\downarrow$	$\downarrow$	$\downarrow$
p_{df}^{ed*}	$\uparrow$	$=$	$\downarrow$
D_d^{ed*}	$\downarrow$, when $\tau_f \geq \frac{(\alpha_{df} - c_f - c_{fm})k_d}{2(A - c_f k_d)}$; $\uparrow$, when $\tau_f < \frac{(\alpha_{df} - c_f - c_{fm})k_d}{2(A - c_f k_d)}$.	$\uparrow$	$\uparrow$
D_{df}^{ed*}	$\downarrow$	$=$	$\uparrow$
Π_F^{ed*}	$\downarrow$	$=$	$\uparrow$
Π_E^{ed*}	$\downarrow$	$=$	$\uparrow$

where $A = (c_{em} + c_c + c_{dd} k_d) - k_e p_e$ as shown in Eq. (2.7).

Overall, under conditions in Eqs. (2.10) and (2.31), the E&DG producer is affected more by changes in τ_f and c_f than the feed producer in the ED model. Our objective is to find how the change in the DG fraction can help the expanding ethanol market with the change in other ingredients' cost level.

As the increase of c_f, both producers lose profit. Moreover, the E&DG producer loses more profit than the feed producer because the leader has twice as much profit loss as the follower.

With conditions in Eqs. (2.10) and (2.31), as an increase in the DG fraction τ_f

1) when $c_f k_d < \mathrm{A}$ in Case 1, the E&DG producer has the profit loss more than the feed producer does. The feed producer will prefer to have the DG fraction as small as possible and the E&DG producer will sell DG as less as possible. Both producers have the maximum profit with a small amount of DG by a small DG fraction, which is not helpful to the ethanol market since the amount of ethanol is small;
2) when $c_f k_d = \mathrm{A}$ in Case 2, both producers have no profit loss. Both producers can more quantity of DG produced with the higher DG fraction even though none of them has more benefit. the other ingredients' cost is equal to a constant, we will not study Case 2;
3) when $c_f k_d > \mathrm{A}$ in Case 3, the E&DG producer benefits more than the feed producer does. Both producers prefer to have the higher DG fraction in order for the higher profit, which results in the higher quantity of DG as well as ethanol. Under this case, it boosts the expanding ethanol market.

In order to show the numerical example in the following section, Case 3: $c_f k_d > \mathrm{A}$ with the conditions $0 < \alpha_{df} - c_f - c_{fm}$ in Eq. (2.10), and $-(\alpha_{df} - c_{fm}) k_d < \mathrm{A} < (\alpha_{df} - c_{fm}) k_d$ inEq. (2.31), is used for the succeeding study.

3. THE CENTRALLY COORDINATED (CC) MODEL

Centrally Coordinated (CC) model is introduced as a benchmark to the ED model. The E&DG producer, together with the feed producer, form a group as the CC model, to evaluate the performance of the ED model. In the CC model, two producers are interdependent and act as a group to maximize the total profit. Jeuland and Shugan [20] pointed out that the total profit is maximized and channel members have the most profits to divide.

3.1 Optimal solution of the CC model

Within the framework of consolidation, both the E&DG producer and the feed producer act coherently so as to maximize the total profit as shown in Eq. (3.1),

$$\prod_{CC} = R_{df} + R_e - c_{em}Q - c_c Q - c_{dd} D_d - c_f \left(D_{df} - D_d \right) - c_{fm} D_{df} . \tag{3.1}$$

In Eq. (3.1), the first term R_{df} is the revenue from selling feed in the feed market and equals to $p_{df} D_{df}$; the second term R_e is the revenue from selling ethanol in the ethanol market and equals to $p_e D_e$; the third term $c_{em} Q$ is the joint production cost for the E&DG production; the fourth term $c_c Q$ is the cost for corn for the joint production of ethanol and DG; the fifth term $c_{dd} D_d$ is the drying cost for each ton of DG after the joint production; the sixth term $c_f \left(D_{df} - D_d \right)$ is the cost of other ingredients for the feed production; and the final term $c_{fm} D_{df}$ is the total processing cost of the feed production.

Similarly as the ED model, the CC model also faces the demand function of feed $D_{df} = \left(\alpha_{df} - p_{df} \right) / \beta_{df}$ from the feed market. The central decision-maker in the CC model only sets the feed price p_{df} as the decision variable according to the assumption **A5**. After substituting Eqs. (2.1), (2.2) and (2.3) into Eq. (3.2), the central decision-maker in the CC model maximizes the profit function with respect to p_{df} as shown in Equation (3.2),

$$\underset{p_{df}}{\text{Max}} \quad \Pi_{CC} = \left(p_{df} + \frac{k_e p_e \tau_f}{k_d} - \frac{c_{em}\tau_f}{k_d} - \frac{c_c \tau_f}{k_d} - c_{dd}\tau_f - c_f\left(1-\tau_f\right) - c_{fm} \right) \frac{\alpha_{df} - p_{df}}{\beta_{df}}. \tag{3.2}$$

The concavity of the profit function of central decision-maker in p_{df} is guaranteed by the second-order sufficient condition:

$$\partial^2 \Pi_{CC} / \partial p_{df}^{\ 2} = -2/\beta_{df} < 0. \tag{3.3}$$

Hence, the first-order necessary condition of Eq. (3.2) is the optimal condition of the CC model, as shown in Eq. (3.4),

$$\frac{\partial \Pi_{CC}}{\partial p_{df}} = \frac{\left(\alpha_{df} + c_f\left(1-\tau_f\right) + c_{fm}\right)k_d + \mathrm{A}\tau_f - 2k_d p_{df}}{\beta_{df} k_d} = 0. \tag{3.4}$$

where $\mathrm{A} = \left(c_{em} + c_c + c_{dd}k_d\right) - k_e p_e$ as in Eq. (2.7).

Then, the optimal feed price $p_{df}^{\ cc*}$ shown in Eq. (3.5) is obtained from Eq. (3.4), where the superscript cc represents the CC model:

$$p_{df}^{\ cc*} = \frac{\left(\alpha_{df} + c_f\left(1-\tau_f\right) + c_{fm}\right)k_d + \mathrm{A}\tau_f}{2k_d}. \tag{3.5}$$

The price of feed, $p_{df}^{\ cc*}$ also is positive since the condition in (3.6) is satisfied due to the condition of $-\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d < \mathrm{A}\tau_f < \left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d$ from Eq.(2.29),

$$\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d + \mathrm{A}\tau_f > 2c_f\left(1-\tau_f\right) + 2c_{fm} > 0, \tag{3.6}$$

where c_f, c_{fm} are positive and $\tau_f \in (0,1)$ in the assumption **A4**.

The corresponding optimal solution of the CC model in D_{df}^{cc*}, D_{d}^{cc*}, Q^{cc*}, D_{e}^{cc*}, p_{df}^{cc*}, and $\prod_{CC}^{*}$ can also be obatined in a straightforward manner as shown in Table 3.1.

Table 3.1 The optimal solution of the CC model

D_{df}^{cc*}	$\dfrac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d}$
D_{d}^{cc*}	$\dfrac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d}\tau_f$
Q^{cc*}	$\dfrac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d^{2}}\tau_f$
D_{e}^{cc*}	$\dfrac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d^{2}}k_e\tau_f$
p_{df}^{cc*}	$\dfrac{\left(\alpha_{df}+c_f\left(1-\tau_f\right)+c_{fm}\right)k_d+\mathrm{A}\tau_f}{2k_d}$
$\prod_{CC}^{*}$	$\dfrac{\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)^2}{4\beta_{df}k_d^{2}}$

In Table 3.1, $\mathrm{A}=\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e$ as in Eq. (2.7).

Next, a comparison between the ED model and the CC model is shown in the following section under conditions in Eqs. (2.10) and (2.31).

3.2 The comparison between the ED model and the CC model

The solutions in the ED model and the CC model are listed in the Table 3.2.

Table 3.2 The comparison between the ED model and the CC model

	The ED model	The CC model
D_{df}^{*}	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{4k_d\beta_{df}}$	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d}$
D_d^{*}	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{4k_d\beta_{df}}\tau_f$	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d}\tau_f$
Q^{*}	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{4k_d^{2}\beta_{df}}\tau_f$	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d^{2}}\tau_f$
D_e^{*}	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{4k_d^{2}\beta_{df}}k_e\tau_f$	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f}{2\beta_{df}k_d^{2}}k_e\tau_f$
p_{df}^{*}	$\frac{\left(4\alpha_{df}-\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)\right)k_d+\mathrm{A}\tau_f}{4k_d}$	$\frac{\left(\alpha_{df}+c_f\left(1-\tau_f\right)+c_{fm}\right)k_d+\mathrm{A}\tau_f}{2k_d}$
p_d^{*}	$\frac{\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d+\mathrm{A}\tau_f}{2k_d\tau_f}$	NA
Π_E^{*}	$\frac{\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)^2}{8k_d^{2}\beta_{df}}$	NA
Π_F^{*}	$\frac{\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)^2}{16k_d^{2}\beta_{df}}$	NA
Π_{CC}^{*}	$\frac{3\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)^2}{16k_d^{2}\beta_{df}}$	$\frac{\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)^2}{4\beta_{df}k_d^{2}}$

In Table 3.2, $\mathrm{A}=\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e$ as in Eq. (2.7).

With conditions in Eqs. (2.10) and (2.31), the solutions under the ED model and the CC model are existed simultaneously. We note that,

$$p_{df}^{\ cc*} < p_{df}^{\ ed*}, D_{df}^{\ cc*} = 2D_{df}^{\ ed*}. \tag{3.7}$$

That is, the central decision-maker provides a lower p_{df} and twice more quantity of feed D_{df} than the ED model. According to the conversions in Eqs. (2.1), (2.2) and (2.3), the central decision-maker provides twice more quantity of DG and twice more quantity of ethanol from twice more quantity of corn than the ED model,

$$D_d^{\ cc*} = 2D_d^{\ ed*}, Q^{cc*} = 2Q^{ed*}, D_e^{\ cc*} = 2D_e^{\ ed*}. \tag{3.8}$$

Nowadays, the corn ethanol industry is expanding because of the stimulation. Thus the CC model is better than the ED model to provide more quantity of ethanol to the ethanol market, in which the lower feed price is provided to the feed market than the ED model.

$$\prod_{CC}{}^{*} > \left(\prod_E{}^{ed*} + \prod_F{}^{ed*}\right). \tag{3.9}$$

The CC model has the higher total profit than the ED model. And, the supply chain performance of the ED model is $\left(\prod_E{}^{ed*} + \prod_F{}^{ed*}\right) / \prod_{CC}{}^{*} = 0.75$, which is to reflect the capability of obtaining profit through the ratio of the total profit over the total optimal profit in the CC model [32]. Additionally, channel coordination is the economic incentive for producers to have the better performance.

In the ED model, the E&DG producer has the performance $\prod_E{}^{ed*} / \prod_{CC}{}^{*} = 0.50$, and the feed producer has the performance $\prod_F{}^{ed*} / \prod_{CC}{}^{*} = 0.25$.

$\prod_{CC}{}^{*} = TR^{cc*} - TC^{cc*} > \prod_E{}^{ed*} + \prod_F{}^{ed*} = TR^{ed*} - TC^{ed*}$, since the increased total revenue is higher than the increased total cost when the CC model is compared to the ED model, where the total revenue in the CC model is higher than that in the ED model, $\Delta TR = TR^{cc*} - TR^{ed*} > 0$, and the total cost in the CC model is higher than that in the ED model, $\Delta TC = TC^{cc*} - TC^{ed*} > 0$.

Moreover, the revenue in selling ethanol in the CC model is twice as much as that in the ED model because the quantity of ethanol in the CC model is twice more than that in the ED model.

$$R_e^{cc*} = p_e D_e^{cc*} = 2R_e^{ed*} = 2p_e D_e^{ed*}. \tag{3.10}$$

The revenue from selling feed in the CC model as well as in the ED model is $R_{df}^{cc*} = p_{df}^{cc*} D_{df}^{cc*}, R_{df}^{ed*} = p_{df}^{ed*} D_{df}^{ed*}$, respectively. In addition, the ratio between these two models about the revenue in selling feed is following.

$$\frac{R_{df}^{ed*}}{R_{df}^{cc*}} = \frac{p_{df}^{ed*} D_{df}^{ed*}}{p_{df}^{cc*} D_{df}^{cc*}} = \frac{1}{2} \frac{4\alpha_{df} - \left(\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f\right)}{4\alpha_{df} - 2\left(\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f\right)} \geq \frac{1}{2}, \tag{3.11}$$

where $\left(\alpha_{df} - c_f\left(1-\tau_f\right) - c_{fm}\right)k_d - \mathrm{A}\tau_f > 0$ from Eq. (2.29).

For the expenses, the CC model is twice than the ED model in the total joint production cost, other ingredients' cost, and the processing cost, respectively.

$$\begin{aligned} &\left(c_{em} + c_c + c_{dd}k_d\right)Q^{cc*} = 2\left(c_{em} + c_c + c_{dd}k_d\right)Q^{ed*} \\ &c_f\left(D_{df}^{cc*} - D_d^{cc*}\right) = 2c_f\left(D_{df}^{ed*} - D_d^{ed*}\right) \\ &c_{fm}D_{df}^{cc*} = 2c_{fm}D_{df}^{ed*} \end{aligned} \tag{3.12}$$

Overall, after the comparison between the ED model and the CC model, both producers would like to coordinate in the same group to split the total profit, due to a maximum total profit, where the increased total revenue is higher than the increased total cost. Moreover, from Eq. (3.8), the E&DG producer can provide more ethanol to the expanding ethanol market because of the lower feed price. Based on these conclusions, government might want to facilitate the sharing of supply chain profit by subsidy, etc. to promote higher production of ethanol.

3.2.1 The analysis of total profit with respect to the DG fraction τ_f

$$\frac{d\prod_{CC}^{*}}{d\tau_f} = \frac{-\left(\mathrm{A}-c_f k_d\right)\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)}{2k_d^{\,2}\beta_{df}} \tag{3.13}$$

$$\frac{d\left(\prod_E^{ed*}+\prod_F^{ed*}\right)}{d\tau_f} = \frac{-3\left(\mathrm{A}-c_f k_d\right)\left(\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f\right)}{8k_d^{\,2}\beta_{df}} \tag{3.14}$$

where $\mathrm{A}=\left(c_{em}+c_c+c_{dd}k_d\right)-k_e p_e$ as shown in Eq. (2.7), and the term $\left(\alpha_{df}-c_f\left(1-\tau_f\right)-c_{fm}\right)k_d-\mathrm{A}\tau_f>0$ from Eq. (2.29).

We noticed that $d\left(\prod_E^{ed*}+\prod_F^{ed*}\right)/d\tau_f=(3/4)d\prod_{CC}^{*}/\left(d\tau_f\right)$. As the same analysis from Table 2.5, as the DG fraction τ_f increases,

1) when $\mathrm{A}-c_f k_d>0$, $\prod_{CC}^{*}$ and $\left(\prod_E^{ed*}+\prod_F^{ed*}\right)$ keep decreasing;

2) when $\mathrm{A}-c_f k_d=0$, $\prod_{CC}^{*}$ and $\left(\prod_E^{ed*}+\prod_F^{ed*}\right)$ do not change;

3) when $\mathrm{A}-c_f k_d<0$, $\prod_{CC}^{*}$ and $\left(\prod_E^{ed*}+\prod_F^{ed*}\right)$ increase.

4. THE E&DG PRODUCER-DRIVEN STACKELBERG MODEL WITH A QUADRATIC UNIT JOINT PRODUCTION COST (EDQ)

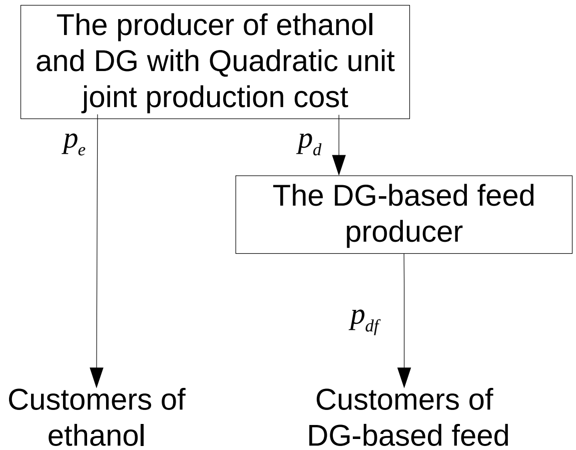


Figure 4.1 Configuration of the EDQ model

In this section, the E&DG Producer-Stackelberg Model with quadratic unit joint production cost in the E&DG producer (EDQ) in Figure 4.1 is proposed according the following assumptions.

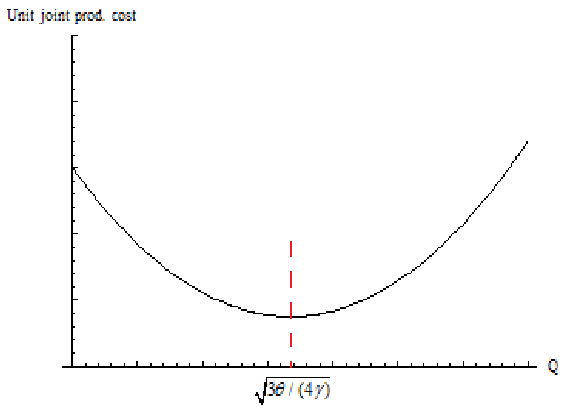
A8. *The unit joint production cost to process one unit of corn is a quadratic function about the processed quantity of corn. And, the total joint production cost is a monotonic increasing polynomial function with respect to the processed quantity of corn.*

$$\hat{C}_{EM}(Q) = \theta Q - \sqrt{3\gamma\theta}Q^2 + \gamma Q^3 \qquad (4.1)$$

From several literatures there exists the quadratic unit production cost function $\hat{C}_{EM}(Q)/Q = \theta - \omega Q + \gamma Q^2$ which has the similar curve shown in Figure 4.2, where the unit joint production cost decrease when $Q \in (0, \omega/(2\gamma)]$ and increases when $Q \in (\omega/(2\gamma), \infty]$.

Thus, the total joint production cost is $\hat{C}_{EM}(Q)=\theta Q-\omega Q^2+\gamma Q^3$. In order to keep the monotonic increasing, there exists $\omega=\sqrt{3\gamma\theta}$ since there is only one Q's value such that

$$\frac{d\hat{C}_{EM}(Q)}{dQ}=\theta-2\omega Q+3\gamma Q^2=0 \tag{4.2}$$



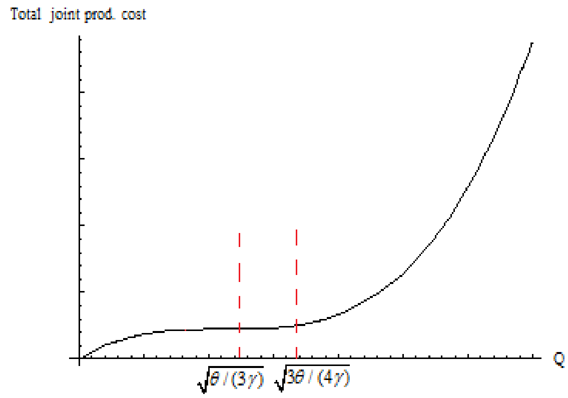


Figure 4.2 Unit joint production cost

Figure 4.3 Total joint production cost

For the E&DG producer, Figure 4.2 is the unit joint production cost and Figure 4.3 is the total joint production cost.

New Parameters

$\hat{C}_{EM}(Q)$ $\hat{C}_{EM}(Q)=\theta Q-\sqrt{3\gamma\theta}Q^2+\gamma Q^3$, the joint production cost of processing one ton of corn (\$/ton), where $\theta>0,\gamma>0$

In this section, a detailed formulation and analysis about a Stackelberg model are given between the E&DG producer and the feed producer, where the former is the Stackelberg leader and the latter is the Stackelberg follower [41]. In the following "EDQ" is denoted as the E&DG producer-Driven Stackelberg model where the E&DG producer with a quadratic unit joint product cost being the leader.

4.1 The feed producer's profit maximization problem

In order to determine the Stackelberg equilibrium by backward induction, we firstly solve the feed producer's profit problem when the E&DG producer's decision variable p_d is given.p_{eg}, p_d, Q In this section, we describe the profit maximization problem of the feed producer. She only purchases DG from the E&DG producer and then sells feed to the feed market.

$\prod_F(p_{df})\prod_F(p_{df})$ is the objective profit function of the feed producer. In the objective function, the feed producer as the follower assumes p_d given, and decides on p_{df} as below:

$$\underset{p_{df}}{\text{Max}} \quad \prod_F(p_{df}) = p_{df}D_{df} - \bar{p}_d D_{df}\tau_f - c_{fm}D_{df} - c_f\left(D_{df} - D_d\right) \tag{4.3}$$

In Eq. (4.3), $\prod_F$, is the objective profit function of the feed producer whose profit is equal to total revenue from selling feed minus her cost. The first term R_{df} is the revenue from selling feed to the customers in the feed market, and equals to $p_{df}D_{df}$; the second term $p_d D_d$ is the cost of purchasing DG from the E&DG producer, the third term $c_{fm}D_{df}$ is the processing cost of DG-based feed production, and the last one, $c_f\left(D_{df} - D_d\right)$ is the cost of other ingredients for producing one unit of feed, and equals to $c_f\left(1-\tau_f\right)D_{df}$.

4.1.1 The standardization of the problem of the feed producer

In this section, we standardize the nonlinear objective functions of the feed producer. Through substituting the demand function into this problem, the standard minimum problem of the feed producer is shown as following,

$$\underset{p_{df}}{\text{Max}} \quad \prod_F(p_{df}) = p_{df}D_{df} - \bar{p}_d D_{df}\tau_f - c_{fm}D_{df} - c_f\left(1-\tau_f\right)D_{df}. \tag{4.4}$$

4.1.2 The best response function of the feed producer

Knowing the given p_d from the E&DG producer, it is easy to verify that this standardized objective function of the feed producer is convex in the price of feed p_{df} , by $\partial^2 \prod_F / \partial p_{df}{}^2 = -2/\beta_{df} > 0$. By the first order necessary condition setting $\partial \prod_F / \partial p_{df} = 0$, the unique best response function of the feed producer is $\hat{p}_{df}(p_d) = \left(\alpha_{df} + c_{fm} + c_f\left(1-\tau_f\right) + \tau_f p_d\right)/2$. After substituting the best response function into $D_d = \tau_f D_{df}$, the quantity of DG purchased by the feed producer can be represented as $D_d = \tau_f D_{df} = \tau_f\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right) - \tau_f p_d\right)/\left(2\beta_{df}\right)$.

$$\hat{p}_{df}(p_d): \qquad \hat{p}_{df}(\overline{p}_d) = \left(\alpha_{df} + c_{fm} + c_f\left(1-\tau_f\right) + \tau_f \overline{p}_d\right)/2 \tag{4.5}$$

4.2 The E&DG producer's profit maximization problem

In this section, we describe the E&DG producer's profit maximization problem. This is the second step in determining the Stackelberg equilibrium. In this section, the maximization profit problem of the E&DG producer is described as follows,

$$\prod_E(p_d) = p_e D_e + p_d D_d - \hat{C}_{EM}(Q) - c_c Q - c_{dd} D_d \tag{4.6}$$

$\prod_E(p_d)$, the objective profit function of the E&DG producer, is a straight-forward algebraic statement which profit is equal to his revenues less his cost. The E&DG producer as the leader decides on p_d and assumes p_{df} given. Because he is the price-taker to ethanol, the selling quantity of ethanol is $k_e Q$.

$$\underset{p_d}{\text{Max}} \quad \prod_E(p_d) = p_e D_e + p_d D_d - \hat{C}_{EM}(Q) - c_c Q - c_{dd} D_d \tag{4.7}$$

where $Q=\tau_f D_{df}/k_d$. In the objective function from Eq. (4.7), the term $p_e D_e = p_e k_e Q$ is the revenue from selling ethanol; the second term $p_d D_d = p_d \tau_f D_{df}$ is the revenue from selling DG to the feed producer; the third term $\hat{C}_{EM}(Q)$ is the joint production cost; the fourth term $c_c Q$ is the cost of corn; the fifth term $c_{dd} D_d$ is the drying cost for obtaining the amount of DG, D_d.

4.2.1 The standardization of the problem of the E&DG producer

In this section, we standardize the nonlinear objective function for the E&DG producer. Through substituting the demand function, the standardized minimum problem of the E&DG producer is shown as following,

$$\begin{aligned}\underset{p_d}{\text{Max}}\quad \prod\nolimits_E(p_d) = {} & p_e k_e \tau_f \hat{D}_{df}(p_{df})/k_d + p_d \tau_f \hat{D}_{df}(p_{df}) \\ & -\hat{C}_{EM}(\hat{D}_{df}(p_{df}))\tau_f/k_d - c_c \tau_f \hat{D}_{df}(p_{df})/k_d - c_{dd}\tau_f \hat{D}_{df}(p_{df})\end{aligned} \tag{4.8}$$

where $\hat{C}_{EM}(\hat{D}_{df}(p_{df})) = \theta\tau_f \hat{D}_{df}(p_{df})/k_d - \sqrt{3\gamma\theta}\left(\tau_f \hat{D}_{df}(p_{df})/k_d\right)^2 + \gamma\left(\tau_f \hat{D}_{df}(p_{df})/k_d\right)^3$,

$\hat{D}_{df}(p_{df}) = \left(\alpha_{df} - p_{df}\right)/\beta_{df}$.

The best response function $\hat{p}_{df}(\overline{p}_d)$ as a function of p_d in Eq. (4.5) is substituted into Eq. (4.8). In order to have the concavity of the profit function of the E&DG producer in p_d, the second-order sufficient condition should be less than zero, $\partial^2 \prod_E/\partial p_d^2 < 0$:

$$\partial^2 \prod\nolimits_E/\partial p_d^2 = \frac{-4k_d^3\beta_{df}^2\tau_f^2 + 2\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^4 - 3\gamma\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right) - p_d\tau_f\right)\tau_f^5}{4k_d^3\beta_{df}^3} < 0 \tag{4.9}$$

A9. *The E&DG producer has a concavity profit function in the DG price.*

Therefore, the DG price is

$$p_d < \frac{\mathrm{K}}{3\gamma\tau_f^{\ 5}} \tag{4.10}$$

where $\mathrm{K} = 4k_d^{\ 3}\beta_{df}^{\ \ 2}\tau_f - 2\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3} + 3\gamma\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right)\right)\tau_f^{\ 4}$.

Eq. (4.11), the first-order necessary condition of Eq. (4.8) in p_d, is the equilibrium condition of the E&DG producer.

$$\frac{\partial \Pi_E}{\partial p_d} = 0 \tag{4.11}$$

From Eq. (4.11), there exist two feasible solutions of the DG price.

$$p_d^{\ edq1} = \frac{\mathrm{K} - 2\Gamma}{3\gamma\tau_f^{\ 5}} \tag{4.12}$$

$$p_d^{\ edq2} = \frac{\mathrm{K} + 2\Gamma}{3\gamma\tau_f^{\ 5}} \tag{4.13}$$

where $\mathrm{K} = 4k_d^{\ 3}\beta_{df}^{\ \ 2}\tau_f - 2\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3} + 3\gamma\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right)\right)\tau_f^{\ 4}$,

$\Gamma = k_d\beta_{df}\tau_f\sqrt{4k_d^{\ 4}\beta_{df}^{\ \ 2} - 4\sqrt{3\gamma\theta}k_d^{\ 2}\beta_{df}\tau_f^{\ 2} + 3\gamma\tau_f^{\ 3}\left(k_d\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right)\right) + \left(k_e p_e - c_c - c_{dd}k_d\right)\tau_f\right)}$.

After separately substituting the DG prices in Eq. (4.12) and (4.13) into the best response function in Eq. (4.5), the feed prices are shown in Eqs. (4.14) and (4.15),

$$p_{df}^{\ \ edq1} = \frac{2k_d^{\ 3}\beta_{df}^{\ \ 2}\tau_f - \sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3} + 3\gamma\alpha_{df}\tau_f^{\ 4} - \Gamma}{3\gamma\tau_f^{\ 4}} \tag{4.14}$$

$$p_{df}^{\ \ edq2} = \frac{2k_d^{\ 3}\beta_{df}^{\ \ 2}\tau_f - \sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3} + 3\gamma\alpha_{df}\tau_f^{\ 4} + \Gamma}{3\gamma\tau_f^{\ 4}} \tag{4.15}$$

where

$\Gamma = k_d\beta_{df}\tau_f\sqrt{4k_d^{\ 4}\beta_{df}^{\ \ 2} - 4\sqrt{3\gamma\theta}k_d^{\ 2}\beta_{df}\tau_f^{\ 2} + 3\gamma\tau_f^{\ 3}\left(k_d\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right)\right) + \left(k_e p_e - c_c - c_{dd}k_d\right)\tau_f\right)}$.

In the assumption **A1**, the price of DG as well as the feed is positive.

1) $\mathrm{K}-2\Gamma>0$ and $\mathrm{K}+2\Gamma>0$

Eqs. (4.12) and (4.13) are positive values when $\mathrm{K}-2\Gamma>0$ and $\mathrm{K}+2\Gamma>0$. Comparing with these two solutions, we find that the E&DG producer has higher profit when he selects the DG price p_d^{edq1} from Eq.(4.16).

$$\Pi_E^{edq1}-\Pi_E^{edq2}=\frac{4\Gamma^3}{27\gamma^2 k_d^{\ 3}\beta_{df}^{\ 3}\tau_f^{\ 9}}\geq 0 \tag{4.16}$$

where Γ is positive.

When $\mathrm{K}-2\Gamma>0$ and $\mathrm{K}+2\Gamma>0$, the equilibrium price of DG in the EDQ model is shown in Eq. (4.17) where the superscript es denotes the ED model and * is the designation of optimality.

$$p_d^{edq*}=\frac{\mathrm{K}-2\Gamma}{3\gamma\tau_f^{\ 5}} \tag{4.17}$$

where $\mathrm{K}=4k_d^{\ 3}\beta_{df}^{\ 2}\tau_f-2\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3}+3\gamma\left(\alpha_{df}-c_{fm}-c_f\left(1-\tau_f\right)\right)\tau_f^{\ 4}$,

$$\Gamma=k_d\beta_{df}\tau_f\sqrt{4k_d^{\ 4}\beta_{df}^{\ 2}-4\sqrt{3\gamma\theta}k_d^{\ 2}\beta_{df}\tau_f^{\ 2}+3\gamma\tau_f^{\ 3}\left(k_d\left(\alpha_{df}-c_{fm}-c_f\left(1-\tau_f\right)\right)+\left(k_e p_e-c_c-c_{dd}k_d\right)\tau_f\right)}.$$

After substituting the equilibrium DG price into the best response function in Eq.(4.5), the equilibrium feed price is,

$$p_{df}^{edq*}=\frac{2k_d^{\ 3}\beta_{df}^{\ 2}\tau_f-\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3}+3\gamma\alpha_{df}\tau_f^{\ 4}-\Gamma}{3\gamma\tau_f^{\ 4}} \tag{4.18}$$

where

$$\Gamma=k_d\beta_{df}\tau_f\sqrt{4k_d^{\ 4}\beta_{df}^{\ 2}-4\sqrt{3\gamma\theta}k_d^{\ 2}\beta_{df}\tau_f^{\ 2}+3\gamma\tau_f^{\ 3}\left(k_d\left(\alpha_{df}-c_{fm}-c_f\left(1-\tau_f\right)\right)+\left(k_e p_e-c_c-c_{dd}k_d\right)\tau_f\right)}.$$

Table 4.1 shows the equilibrium solution corresponding to the quantities and the profits in the supply chain, where all values are positive and the superscript *edq* represents the EDQ model.

Table 4.1 The equilibrium solution of the ED model when $K-2\Gamma>0$ and $K+2\Gamma>0$

D_{df}^{edq*}	$\frac{-2k_d^{\ 3}\beta_{df}^{\ 2}\tau_f+\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3}+\Gamma}{3\gamma\beta_{df}\tau_f^{\ 4}}$
D_d^{edq*}	$\frac{-2k_d^{\ 3}\beta_{df}^{\ 2}\tau_f+\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3}+\Gamma}{3\gamma\beta_{df}\tau_f^{\ 3}}$
Q^{edq*}	$\frac{-2k_d^{\ 3}\beta_{df}^{\ 2}\tau_f+\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3}+\Gamma}{3k_d\gamma\beta_{df}\tau_f^{\ 3}}$
D_e^{edq*}	$k_e\frac{-2k_d^{\ 3}\beta_{df}^{\ 2}\tau_f+\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3}+\Gamma}{3k_d\gamma\beta_{df}\tau_f^{\ 3}}$
p_{df}^{edq*}	$\frac{2k_d^{\ 3}\beta_{df}^{\ 2}\tau_f-\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3}+3\gamma\alpha_{df}\tau_f^{\ 4}-\Gamma}{3\gamma\tau_f^{\ 4}}$
p_d^{edq*}	$\frac{K-2\Gamma}{3\gamma\tau_f^{\ 5}}$
$\prod_E^{edq*}$	$\frac{1}{27\gamma^2k_d\beta_{df}\tau_f^7}(-16k_d^7\beta_{df}^4\tau_f+24\sqrt{3}\sqrt{\gamma\theta}k_d^5\beta_{df}^3\tau_f^3-18\gamma k_d^3(\theta-c_c+k_ep_e)\beta_{df}^2\tau_f^5-6\gamma(c_c-k_ep_e)\tau_f^4\Gamma+\sqrt{3}\sqrt{\gamma\theta}k_d^2\beta_{df}\tau_f^2(9\gamma\tau_f^4(-c_{\mathrm{fm}}+\alpha_{df}+c_f(-1+\tau_f)-c_{\mathrm{dd}}\tau_f)-8\Gamma)+2k_d^4\beta_{df}^2(9\gamma\tau_f^4(c_{\mathrm{fm}}-\alpha_{df}-c_f(-1+\tau_f)+c_{\mathrm{dd}}\tau_f)+4\Gamma)-3\gamma k_d\tau_f^3(\sqrt{3}\sqrt{\gamma\theta}(\theta+3c_c-3k_ep_e)\beta_{df}\tau_f^4-2(-c_{\mathrm{fm}}+\alpha_{df}+c_f(-1+\tau_f)-c_{\mathrm{dd}}\tau_f)\Gamma))$
$\prod_F^{edq*}$	$\frac{1}{9\gamma^2\tau_f^7}k_d(8k_d^5\beta_{df}^3\tau_f-8\sqrt{3}\sqrt{\gamma\theta}k_d^3\beta_{df}^2\tau_f^3+3\gamma k_d(\theta-c_c+k_ep_e)\beta_{df}\tau_f^5+2\sqrt{3}\sqrt{\gamma\theta}\tau_f^2\Gamma+k_d^2\beta_{df}(-3\gamma\tau_f^4(c_{\mathrm{fm}}-\alpha_{df}-c_f(-1+\tau_f)+c_{\mathrm{dd}}\tau_f)-4\Gamma))$

where $\mathrm{K} = 4k_d^{\ 3}\beta_{df}^{\ 2}\tau_f - 2\sqrt{3\gamma\theta}k_d\beta_{df}\tau_f^{\ 3} + 3\gamma\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right)\right)\tau_f^{\ 4}$,

$$\Gamma = k_d\beta_{df}\tau_f\sqrt{4k_d^{\ 4}\beta_{df}^{\ 2} - 4\sqrt{3\gamma\theta}k_d^{\ 2}\beta_{df}\tau_f^{\ 2} + 3\gamma\tau_f^{\ 3}\left(k_d\left(\alpha_{df} - c_{fm} - c_f\left(1-\tau_f\right)\right) + \left(k_e p_e - c_c - c_{dd}k_d\right)\tau_f\right)}.$$

4.3 The comparison between the ED model and the EDQ model

1) When $\mathrm{K} - 2\Gamma > 0$ and $\mathrm{K} + 2\Gamma > 0$, the ED model has the equilibrium solution as shown in Table 2.1 and the EDQ model has the equilibrium solution as shown in Table 4.1.

$$\prod_E^{edq*} - \prod_E^{ed*} = \frac{1}{216k_d^2\beta_{df}}(-27(k_d(c_f + c_{fm} - \alpha_{df}) + (c_c + c_{em} + (c_{dd} - c_f)k_d - k_e p_e)\tau_f)^2 +$$
$$\frac{1}{\gamma^2\tau_f^7}8k_d(-16k_d^7\beta_{df}^4\tau_f + 24\sqrt{3}\sqrt{\gamma\theta}k_d^5\beta_{df}^3\tau_f^3 - 18\gamma k_d^3(\theta - c_c + k_e p_e)\beta_{df}^2\tau_f^5 - 6\gamma(c_c - k_e p_e)\tau_f^4\Gamma +$$
$$\sqrt{3}\sqrt{\gamma\theta}k_d^2\beta_{df}\tau_f^2(9\gamma\tau_f^4(-c_{fm} + \alpha_{df} + c_f(-1 + \tau_f) - c_{dd}\tau_f) - 8\Gamma) + 2k_d^4\beta_{df}^2(9\gamma\tau_f^4(c_{fm} - \alpha_{df} - c_f(-1 + \tau_f) +$$
$$c_{dd}\tau_f) + 4\Gamma) - 3\gamma k_d\tau_f^3(\sqrt{3}\sqrt{\gamma\theta}(\theta + 3c_c - 3k_e p_e)\beta_{df}\tau_f^4 - 2(-c_{fm} + \alpha_{df} + c_f(-1 + \tau_f) - c_{dd}\tau_f)\Gamma)))$$

(4.19)

Eq. (4.19) is the profit difference of the E&DG producer between the ED model and the EDQ model.

$$\prod_F^{edq*} - \prod_F^{ed*} = -\frac{(k_d(c_f + c_{fm} - \alpha_{df}) + (c_c + c_{em} + (c_{dd} - c_f)k_d - k_e p_e)\tau_f)^2}{16k_d^2\beta_{df}} + \frac{1}{9\gamma^2\tau_f^7}k_d\left(8k_d^5\beta_{df}^3\tau_f - 8\sqrt{3}\sqrt{\gamma\theta}k_d^3\beta_{df}^2\tau_f^3 +\right.$$
$$\left.3\gamma k_d(\theta - c_c + k_e p_e)\beta_{df}\tau_f^5 + 2\sqrt{3}\sqrt{\gamma\theta}\tau_f^2\Gamma + k_d^2\beta_{df}\left(-3\gamma\tau_f^4\left(c_{fm} - \alpha_{df} - c_f\left(-1 + \tau_f\right) + c_{dd}\tau_f\right) - 4\Gamma\right)\right)$$

(4.20)

Eq. (4.20) is the profit difference of the feed producer between the ED model and the EDQ model.

$$Q^{edq*} - Q^{ed*} = \frac{1}{12k_d^2\beta_{df}\tau_f^3}(3\tau_f^4(k_d(c_f + c_{fm} - \alpha_{df}) + (c_c + c_{em} + (c_{dd} - c_f)k_d - k_e p_e)\tau_f) +$$
$$\frac{4k_d(-2k_d^3\beta_{df}^2\tau_f + \sqrt{3}\sqrt{\gamma\theta}k_d\beta_{df}\tau_f^3 + \Gamma)}{\gamma})$$

(4.21)

Eq. (4.21) is the quantity difference of corn between the ED model and the EDQ model.

$$p_d^{edq*} - p_d^{ed*} = \frac{1}{6\gamma k_d \tau_f^5}(8k_d^4\beta_{\mathrm{df}}^2\tau_f - 4\sqrt{3}\sqrt{\gamma\theta}k_d^2\beta_{\mathrm{df}}\tau_f^3 - 3\gamma(c_c + c_{\mathrm{em}} - k_e p_e)\tau_f^5 + k_d(-3\gamma\tau_f^4(c_{\mathrm{fm}} - \alpha_{\mathrm{df}} - c_f(-1 + \tau_f) + c_{\mathrm{dd}}\tau_f) - 4\Gamma)) \quad (4.22)$$

Eq. (4.22) is the price difference of DG between the ED model and the EDQ model.

$$p_{df}^{edq*} - p_{df}^{ed*} = -\frac{k_d(c_f + c_{\mathrm{fm}} + 3\alpha_{\mathrm{df}}) + (c_c + c_{\mathrm{em}} + (c_{\mathrm{dd}} - c_f)k_d - k_e p_e)\tau_f}{4k_d} - \frac{-2k_d^3\beta_{\mathrm{df}}^2\tau_f + \sqrt{3}\sqrt{\gamma\theta}k_d\beta_{\mathrm{df}}\tau_f^3 - 3\gamma\alpha_{\mathrm{df}}\tau_f^4 + \Gamma}{3\gamma\tau_f^4} \quad (4.23)$$

Eq. (4.23) is the price difference of the DG-based feed between the ED model and the EDQ model.

From the above comparison, it is difficult to find in which model the E&DG producer or the feed producer has the higher profit. Under this situation, it will be presented by a numerical example.

4.4 The comparison of total joint production cost between the ED model and EDQ model

When the equilibrium quantity of corn in the EDQ model is equal to the equilibrium quantity of corn in the ED model, comparing with the objective profit functions in Eq. (4.6) with the case of nonlinear joint production cost and Eq. (2.16) with the case of linear joint production cost, the profit difference of the E&DG producer between Eq. (2.16) and Eq. (4.6) is equal to

$$\hat{C}_{EM}(Q) - c_{em}Q = \theta Q - \sqrt{3\gamma\theta}Q^2 + \gamma Q^3 - c_{em}Q = \left((\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2\right)Q \quad (4.24)$$

Hence, when Eq. (4.24) is greater than zero, the E&DG producer has higher joint production cost in the EDQ model than in the ED model. When Eq. (4.24) is equal to zero, there is no difference in the joint production cost of the E&DG producer in both models. Otherwise, the E&DG producer has lower joint production cost in the EDQ model than in the ED model.

1) If for any positive quantity of corn Q, the joint production cost in the nonlinear case is always bigger than or equal to that in the linear case, $\hat{C}_{EM}(Q) \geq c_{em}Q$ as shown in the Figure 4.4.

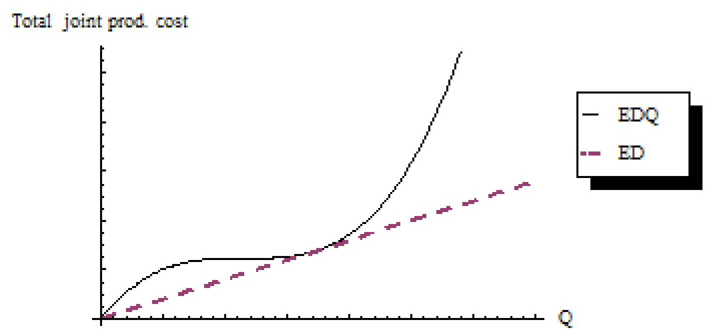


Figure 4.4 Total joint production cost and average joint production cost (Situation 1)

Hence, for any positive Q, the term $(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ in Eq. (4.24) is greater than or equal to zero,

$$(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2 \geq 0 \qquad (4.25)$$

And the derivative of $(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ with respect to Q is,

$$\frac{d\left((\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2\right)}{dQ} = -\sqrt{3\gamma\theta} + 2\gamma Q \qquad (4.26)$$

We find that $d\left((\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2\right)^2 / dQ^2 = 2\gamma > 0$. By setting that Eq. (4.26) equals to zero, the term $(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ achieves the minimal value when

$Q=\sqrt{3\gamma\theta}/(2\gamma)$. As we know from Eq. (4.25), for any positive Q, $(\theta-c_{em})-\sqrt{3\gamma\theta}Q+\gamma Q^2\geq 0$. Therefore, when $Q=\sqrt{3\gamma\theta}/(2\gamma)$,

$$\begin{aligned}(\theta-c_{em})-\sqrt{3\gamma\theta}Q+\gamma Q^2&=(\theta-c_{em})-\sqrt{3\gamma\theta}\sqrt{3\gamma\theta}/(2\gamma)+\gamma\left(\sqrt{3\gamma\theta}/(2\gamma)\right)^2\\&=\frac{1}{4}\theta-c_{em}\geq 0\end{aligned} \tag{4.27}$$

When the unit joint production cost in the linear case is less than $\theta/4$, Eq. (4.27) is greater than or equal to zero for any positive Q. That is, the E&DG producer has higher joint production cost in the EDQ model than in the ED model.

$$c_{em}\leq\theta/4 \tag{4.28}$$

2) Then, when $c_{em}>\theta/4$, for any positive quantity of corn Q, $\hat{C}_{EM}(Q)$ and $c_{em}Q$ have two intersections in the Figure 4.5 and there is one intersections in the Figure 4.6.

$$c_{em}>\theta/4 \tag{4.29}$$

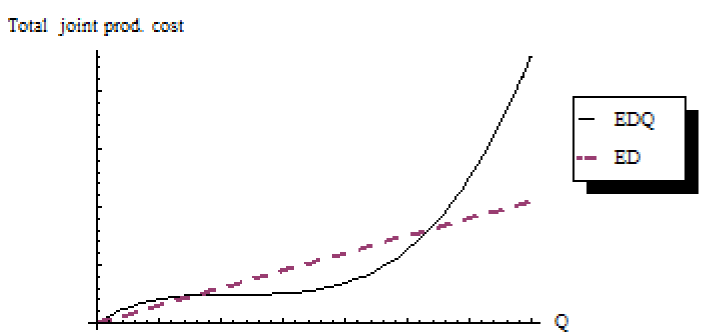


Figure 4.5 Total joint production cost and average joint production cost (Situation 2)

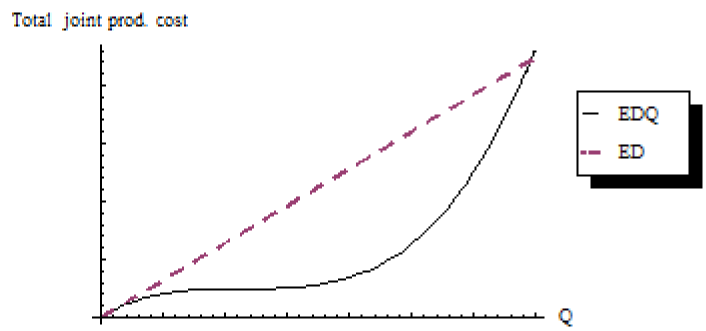


Figure 4.6 Total joint production cost and average joint production cost (Situation 3)

Hence, for any positive Q, the intersection can be obtained from setting the term $(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ in Eq. (4.24) is equal to zero,

$$(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2 = 0 \quad (4.30)$$

Then, two solutions of the quantity of corn Q from Eq. (4.30) are,

$$Q_1 = \frac{\sqrt{\theta} - \sqrt{c_{em}}}{\sqrt{3\gamma}}, \quad (4.31)$$

$$Q_2 = \frac{\sqrt{\theta} + \sqrt{c_{em}}}{\sqrt{3\gamma}}, \quad (4.32)$$

where, $Q_1 < Q_2$.

In order to guarantee that there are two intersections as shown in the Figure 4.5, Eqs.(4.31) and (4.32) should be positive. Hence,

$$\theta > c_{em} > \theta / 4 \quad (4.33)$$

a. When $Q < Q_1 = \left(\sqrt{\theta} - \sqrt{c_{em}}\right) / \sqrt{3\gamma}$, the term $(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ in Eq. (4.24) is greater than zero so that Eq. (4.24) is greater than zero for any positive Q. That is, the E&DG producer has higher joint production cost in the EDQ model than in the ED model.

Proof: for any positive $Q < Q_1 = \left(\sqrt{\theta} - \sqrt{c_{em}}\right) / \sqrt{3\gamma}$,

$$\begin{aligned}
&\left(\theta - c_{em}\right) - \sqrt{3\gamma\theta}Q + \gamma Q^2 = [(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2] - [(\theta - c_{em}) - \sqrt{3\gamma\theta}Q_1 + \gamma Q_1^{\,2}] \\
&= \sqrt{3\gamma\theta}(Q_1 - Q) - \gamma(Q_1^{\,2} - Q^2) = [\sqrt{3\gamma\theta} - \gamma(Q_1 + Q)](Q_1 - Q) \\
&> [\sqrt{3\gamma\theta} - \gamma(2Q_1)](Q_1 - Q) = [\sqrt{3\gamma\theta} - (2\sqrt{\gamma\theta} - 2\sqrt{\gamma c_{em}}) / \sqrt{3}](Q_1 - Q) \\
&= [(\sqrt{\gamma\theta} + 2\sqrt{\gamma c_{em}}) / \sqrt{3}](Q_1 - Q) > 0
\end{aligned} \tag{4.34}$$

b. When $Q > Q_2 = \left(\sqrt{\theta} + \sqrt{c_{em}}\right) / \sqrt{3\gamma}$, the term $\left(\theta - c_{em}\right) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ in Eq. (4.24) is greater than zero so that Eq. (4.24) is greater than zero for any positive Q. That is, the E&DG producer has higher joint production cost in the EDQ model than in the ED model.

Proof: for any positive $Q > Q_2 = \left(\sqrt{\theta} + \sqrt{c_{em}}\right) / \sqrt{3\gamma}$,

$$\begin{aligned}
&\left(\theta - c_{em}\right) - \sqrt{3\gamma\theta}Q + \gamma Q^2 = [(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2] - [(\theta - c_{em}) - \sqrt{3\gamma\theta}Q_2 + \gamma Q_2^{\,2}] \\
&= -\sqrt{3\gamma\theta}(Q - Q_2) + \gamma(Q^2 - Q_2^{\,2}) = [-\sqrt{3\gamma\theta} + \gamma(Q + Q_2)](Q - Q_2) \\
&> [-\sqrt{3\gamma\theta} + \gamma(2Q_2)](Q - Q_2) = [-\sqrt{3\gamma\theta} + (2\sqrt{\gamma\theta} + 2\sqrt{\gamma c_{em}}) / \sqrt{3}](Q - Q_2) \\
&= [(-\sqrt{\gamma\theta} + 2\sqrt{\gamma c_{em}}) / \sqrt{3}](Q - Q_2) > 0
\end{aligned} \tag{4.35}$$

c. Otherwise, when $\left(\sqrt{\theta} - \sqrt{c_{em}}\right) / \sqrt{3\gamma} = Q_1 \le Q \le Q_2 = \left(\sqrt{\theta} + \sqrt{c_{em}}\right) / \sqrt{3\gamma}$, the term $\left(\theta - c_{em}\right) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ in Eq. (4.24) is less than or equal to zero so that Eq. (4.24) is less than or equal to zero for any positive Q. That is, the E&DG producer has lower joint production cost in the EDQ model than in the ED model.

In order to guarantee that there is one intersection as shown in the Figure 4.6, Eq.(4.31) should be non-positive and Eq.(4.32) should be positive. Hence,

$$c_{em} \ge \theta \tag{4.36}$$

a. When $Q > Q_2 = \left(\sqrt{\theta} + \sqrt{c_{em}}\right) / \sqrt{3\gamma}$, the term $\left(\theta - c_{em}\right) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ in Eq. (4.24) is greater than zero so that Eq. (4.24) is greater than zero for any positive Q. That is, the E&DG producer has higher joint production cost in the EDQ model than in the ED model.

Proof: for any positive $Q > Q_2 = \left(\sqrt{\theta} + \sqrt{c_{em}}\right) / \sqrt{3\gamma}$,

$$
\begin{aligned}
&\left(\theta - c_{em}\right) - \sqrt{3\gamma\theta}Q + \gamma Q^2 = [(\theta - c_{em}) - \sqrt{3\gamma\theta}Q + \gamma Q^2] - [(\theta - c_{em}) - \sqrt{3\gamma\theta}Q_2 + \gamma Q_2{}^2] \\
&= -\sqrt{3\gamma\theta}(Q - Q_2) + \gamma(Q^2 - Q_2{}^2) = [-\sqrt{3\gamma\theta} + \gamma(Q + Q_2)](Q - Q_2) \\
&> [-\sqrt{3\gamma\theta} + \gamma(2Q_2)](Q - Q_2) = [-\sqrt{3\gamma\theta} + (2\sqrt{\gamma\theta} + 2\sqrt{\gamma c_{em}}) / \sqrt{3})](Q - Q_2) \\
&= [(-\sqrt{\gamma\theta} + 2\sqrt{\gamma c_{em}}) / \sqrt{3})](Q - Q_2) > 0
\end{aligned}
\qquad (4.37)
$$

b. Otherwise, when $Q > Q_2 = \left(\sqrt{\theta} + \sqrt{c_{em}}\right) / \sqrt{3\gamma}$, the term $\left(\theta - c_{em}\right) - \sqrt{3\gamma\theta}Q + \gamma Q^2$ in Eq. (4.24) is less than or equal to zero so that Eq. (4.24) is less than or equal to zero for any positive Q. That is, the E&DG producer has lower joint production cost in the EDQ model than in the ED model.

5. APPLICATION AND NUMERICAL ANALYSIS

This section is to present the analysis and application of the numerical example for all models. One ton of corn often results in 1/3 ton of ethanol and 1/3 ton of DG in the dry mill processing [1], thus let us assume that the proportion of ethanol as well as DG produced from one unit of corn, $k_e = k_d = 1/3$.

The extension at Iowa State University publicly announce the price of corn, DG, and ethnanol from Oct. 2006 to Sept. 2009 [45]. In Iowa, the price of corn has ranged from $2.33/bushel to $6.84/bushel ($91/ton~$269/ton); the price of DG (with 10% moisture) has ranged from $71/ton ~$196/ton; and the price of ethanol has ranged from $1.42/gallon to $2.80/gallon ($468/ton ~$924/ton) [45]. Hence, let us assume that the price of ethanol $p_e = \$900/\text{ton}$, and the corn cost $c_c = \$180/\text{ton}$. Perrin [46] estimated an average drying cost of $25.80/ton of dry matter in DG. In this paper, we assume that the E&DG producer has the joint production cost of processing one ton of corn $c_{em} = \$80/\text{ton}$ and the drying cost of obtaining one ton of dried DG $c_{dd} = \$60/\text{ton}$. The processing cost of producing one ton of feed is assumed $c_{fm} = \$10/\text{ton}$.

As for the DG fraction, the maximum DG fraction for swine is recommended from 0 to 0.60 [47], appropriate DG fraction for cattle has been achieved at the levels of 0.40 to 0.60 [48],[49]. In the case of the DG fraction for swine and beef, let us assume $\tau_f = 0.40$ [50],[51],[52],[53].

In summary, the parameters in the numerical example except the ethanol price and the other ingredients' cost are:

$$k_e = k_d = 1/3, \alpha_{df} = 180, \beta_{df} = 5\times 10^{-5}, \tau_f = 0.40,$$
$$p_e = \$750/\text{ton}, c_{em} = \$80/\text{ton}, c_c = \$180/\text{ton}, c_{dd} = \$60/\text{ton}, c_{fm} = \$10/\text{ton}.$$

From a survey by Saunder and Rosentrater [54], for the E&DG producer, the average amount of DG is 131,205 tons per year and the median value is 74,000tons per year. Since the amount of DG can indicate the amount of corn for the production, so the corresponding average amount of corn is 393,605 tons per year and the median value is 222,000tons per year [55]. And the average capacity of each E&DG producer in 2008 is 58M gallon of ethanol (520,000tons of corn) from the EPA's record. In this paper, we assume the capacity of the E&DG producer is 450,000 tons of corn (which also can be represented by 150,000 tons of DG or 150,000tons of ethanol) [55].

5.1 Numerical solution of the ES model

As the other ingredients' cost increases, the profit of both producers decreases and the quantity of DG as well as ethanol decrease, which is no helpful to the expanding ethanol market. However, with the higher DG fraction may help to increase the amount of ethanol for the ethanol market and increase the profit of both producers.

For this numerical example, from condition in Eq.(2.10) there exist $c_f < \alpha_{df} - c_{fm} = 170$ and from condition in Eq.(2.31) there is $(\alpha_{df} - c_{fm})k_d = 170/3 > \mathrm{A} = 30 > -(\alpha_{df} - c_{fm})k_d = -170/3$. Therefore, the other ingredients' cost is less than $\$170/\text{ton}$.

5.1.1 The analysis of the ES model with respect to the other ingredients' cost c_f

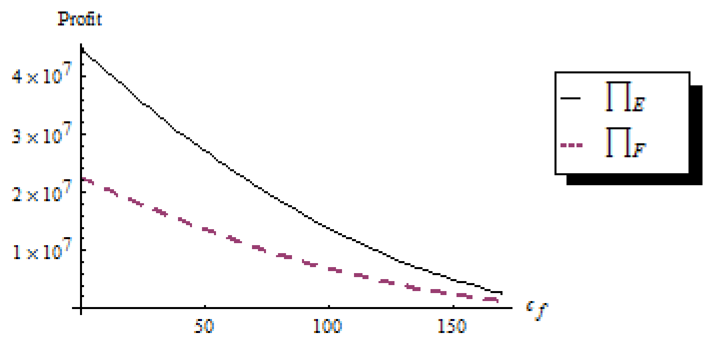


Figure 5.1 The profits of both producers with respect to cost of other ingredients

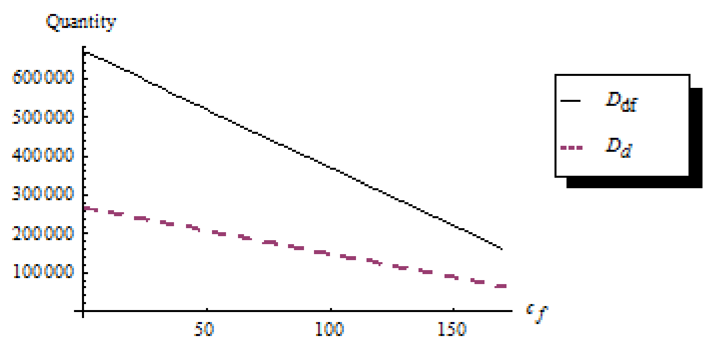


Figure 5.2 The quantity of DG as well as feed with respect to cost of other ingredients

When $\tau_f = 0.40$, as the other ingredients' cost increases, from Figure 5.1 both producers lose profit and from Figure 5.2 the quantity of feed as well as DG decreases. Since Eq. (2.3) shows the fixed ratio relation of quantity between ethanol and DG, the quantity of ethanol deceases simultaneously.

5.1.2 The analysis of the ES model with respect to the DG fraction τ_f

As we know, with conditions in Eqs. (2.10) and (2.31), there are 3 cases: 1) $A > c_f k_d$; 2) $A = c_f k_d$; 3) $A < c_f k_d$. Thus, we will show three cases under different levels of the other ingredients' cost (i.e., forage, alfalfa, and corn), in that the impact of the DG fraction on increasing the quantity of DG as well as ethanol is presented.

Case 1: $A > c_f k_d$, where $A = (c_{em} + c_c + c_{dd} k_d) - k_e p_e$.

Therefore, the other ingredients' cost is $c_f <$ \$90/ton according to $A > c_f k_d$. Let us assume the other ingredients' cost $c_f =$ \$80/ton.

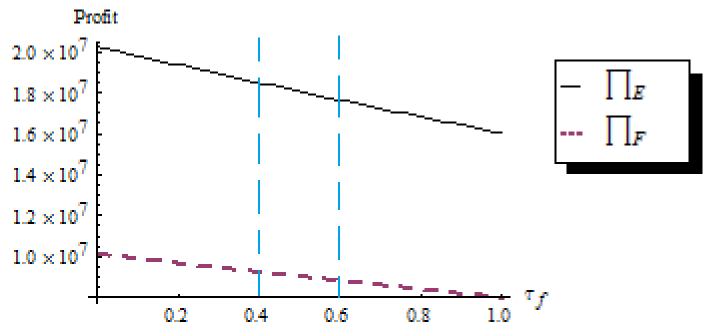


Figure 5.3 The profits of both producers with respect to the DG fraction (Case 1)

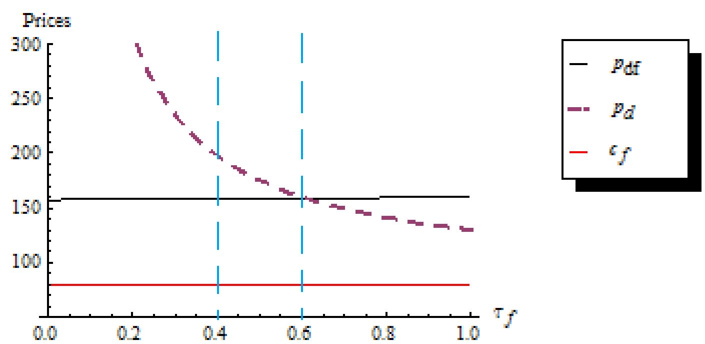


Figure 5.4 The price of DG as well as feed with respect to the DG fraction (Case 1)

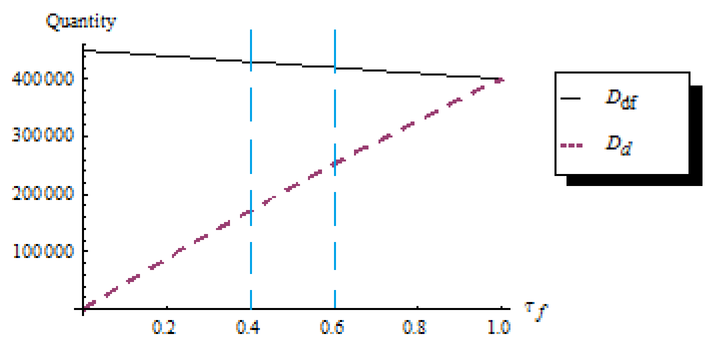


Figure 5.5 The quantity of DG as well as feed with respect to the DG fraction (Case 1)

As the DG fraction increases, we will have the findings from Case 1 as following.

Both producers have the lower profit from Figure 5.3; the price of DG decreases and is higher than other ingredients' cost, and the feed price increases from Figure 5.4; the quantity of feed decreases but the quantity of DG increases from Figure 5.5, since the range of DG fraction is $0 < \tau_f < 1$ from the assumption **A4** and that the quantity of DG increases as the increase of the DG fraction when $\tau_f < (\alpha_{df} - c_f - c_{fm})k_d / (2(\mathrm{A} - c_f k_d)) = 4.5$ from Table 2.5.

a) When $\tau_f \to 1$, there is the highest feed price $p_{df} = \$160/\text{ton}$ which is less than the maximum feed price $\alpha_{df} = \$180/\text{ton}$, thus the quantity of feed decreases to the lowest amount according to the down-slope demand function of feed, but quantity of DG is highest amount. Both producers have the lowest profit.

b) When $\tau_f \to 0$, there is the lowest feed price $\$157/\text{ton}$, thus the quantity of feed achieves the highest amount according to the down-slope demand function of feed. At the same time, the DG price is approaching infinite because the feed producer utilizes extremely small amount of DG that is close to zero for the feed production. The feed producer has the profit from selling more feed and the E&DG producer has the profit from selling extremely small amount of DG with the extremely high DG price.

And usually the DG fraction for all kinds of animal [50],[51],[52] has been ranged from0 to 0.6. So, the E&DG producer has $\prod_E^{ed*}$ ranged in $[\$17,640,000, \$20,250,000)$, and the feed producer has $\prod_F^{ed*}$ ranged in $[\$8,820,000, \$10,125,000)$. The DG price is ranged in

$[\$160/\text{ton},\infty)$ and the feed price is ranged in $(\$157/\text{ton},\$159/\text{ton}]$. And the DG quantity is ranged in $(0\text{ton},252{,}000\text{ton}]$ and the feed quantity is ranged in $[420{,}000\text{ton},450{,}000\text{ton})$.

When given $\tau_f = 0.40$, the E&DG producer has $\prod_E{}^{ed*} = \$18{,}490{,}000$ and the feed producer has $\prod_F{}^{ed*} = \$9{,}245{,}000$. The DG price is $p_d{}^{ed*} = \$197/\text{ton}$ and the feed price is $p_{df}{}^{ed*} = \$158/\text{ton}$. In addition, the DG quantity is $D_d{}^{ed*} = 172{,}000\text{ton}$ and the feed quantity is $D_{df}{}^{ed*} = 430{,}000\text{ton}$.

With the lowest DG fraction $\tau_f \to 0$, both producers have the maximum profit and the quantity of feed is the maximum since the feed price is the lowest. However, the quantity of DG as well as ethanol is the lowest and approaching to zero since the DG price is infinite, which is not helpful for the expanding ethanol market.

Case 2: $\text{A} = c_f k_d$, where $\text{A} = (c_{em} + c_c + c_{dd} k_d) - k_e p_e$.

Therefore, the other ingredients' cost is $c_f = \$90/\text{ton}$ according to $\text{A} = c_f k_d$. Let us assume the other ingredients' cost $c_f = \$90/\text{ton}$.

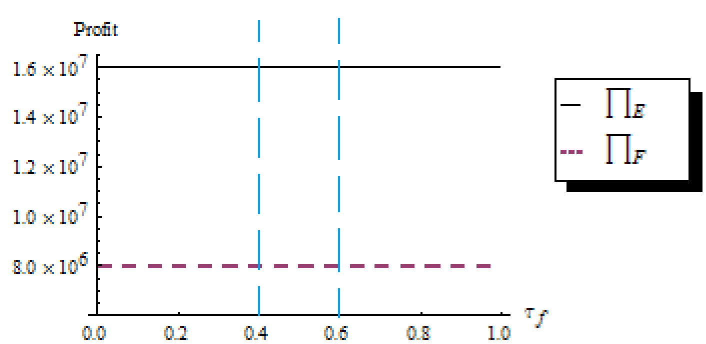


Figure 5.6 The profits of both producers with respect to the DG fraction (Case 2)

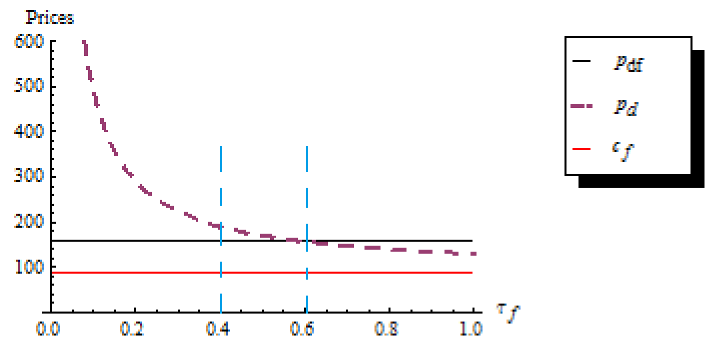


Figure 5.7 The price of DG as well as feed with respect to the DG fraction (Case 2)

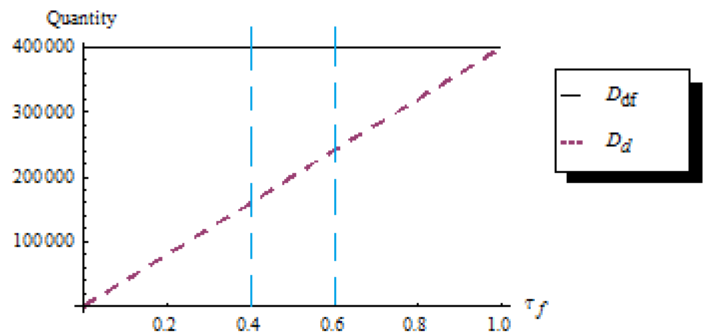


Figure 5.8 The quantity of DG as well as feed with respect to the DG fraction (Case 2)

As the DG fraction increases, we will have the findings as following:

Both producers have the same profit from Figure 5.6; the price of DG decreases and is higher than other ingredients' cost, and the feed price does not change from Figure 5.7; the quantity of feed does not change, however, the quantity of DG keeps increasing from Figure 5.8. The non-change of feed price results in the non-change of quantity of feed according to the down-slope demand function of feed.

a) When $\tau_f \rightarrow 1$, the quantity of DG is approaching the highest 400,000ton by following Eq. (2.1) and the DG price approaches the lowest $130/ton.

b) When $\tau_f \rightarrow 0$, the DG price is approaching infinite because the feed producer utilizes extremely small amount of DG, which is close to zero for the feed production. The feed producer has the profit $\$8,000,000$ and the E&DG producer has the profit $\$16,000,000$.

Under this condition of Case 2, Eq. (2.3) tells that the quantity of ethanol is expanding as the DG fraction increases, even though there is no change in each one's profit. The lower DG price attracts the feed producer to use more DG in the feed.

And usually the DG fraction for all kinds of animal has been ranged in (0,0.6] [50],[51],[52]. Therefore, the E&DG producer has $\prod_E^{ed*}$ is $\$16,000,000$, and the feed producer has $\prod_F^{ed*}$ is $\$8,000,000$. The DG price is ranged in $[\$156/\text{ton}, \infty)$ and the feed price is $\$160/\text{ton}$. And the DG quantity is ranged in (0ton,240,000ton] and the feed quantity is $400{,}000\text{ton}$.

When given $\tau_f = 0.40$, the E&DG producer has $\prod_E^{ed*} = \$16,000,000$ and the feed producer has $\prod_F^{ed*} = \$8,000,000$. The DG price is $p_d^{ed*} = \$190/\text{ton}$ and the feed price is $p_{df}^{ed*} = \$160/\text{ton}$. In addition, the DG quantity is $D_d^{ed*} = 160,000\text{ton}$ and the feed quantity is $D_{df}^{ed*} = 400{,}000\text{ton}$.

With the highest DG fraction $\tau_f \to 1$, both producers do not change the profit and the quantity of feed is constant since the feed price is constant. However, the DG price is the lowest so that the quantity of DG as well as ethanol is the highest, which is helpful for the expanding ethanol market. Under the condition in Case 2, with the highest DG fraction, even there is no change in profit for both producers, the E&DG producer produces the maximum quantity of ethanol.

Case 3: $\mathrm{A} < c_f k_d$, where $\mathrm{A} = (c_{em} + c_c + c_{dd} k_d) - k_e p_e$.

Therefore, the other ingredients' cost is $c_f > \$90/\text{ton}$ according to $A = c_f k_d$. Let us assume the other ingredients' cost $c_f = \$150/\text{ton}$, since Eq. (2.10) requires $c_f < \alpha_{df} - c_{fm} = 170$.

In terms of the impact from the change of DG fraction τ_f, the change of each producer's profit is shown in Figure 5.9; the change of price of DG as well as feed is shown in Figure 5.10; and the change of quantity of DG as well as feed is shown in Figure 5.11.

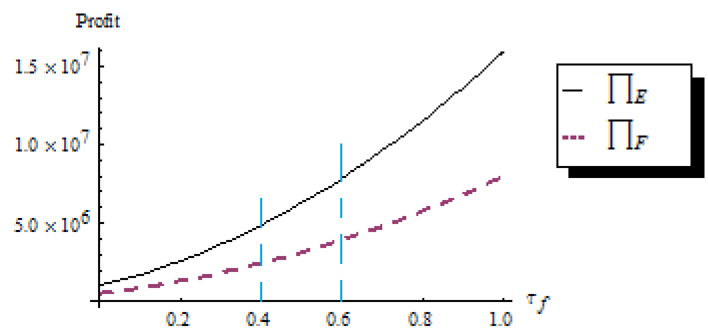


Figure 5.9 The profits of both producers with respect to the DG fraction (Case 3)

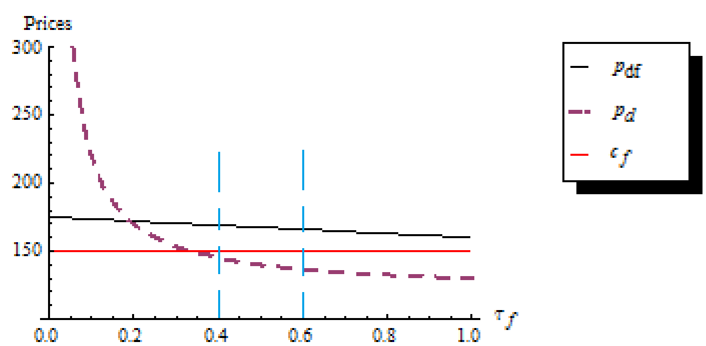


Figure 5.10 The price of DG as well as feed with respect to the DG fraction (Case 3)

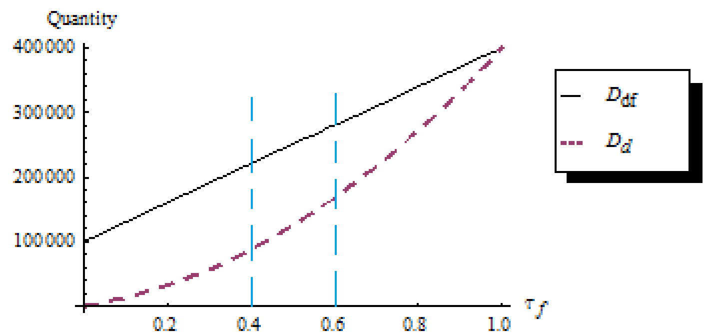


Figure 5.11 The quantity of DG as well as feed with respect to the DG fraction (Case 3)

As the DG fraction increases, we will have the findings from Case 3 as following:

Both producers have the higher profit from Figure 5.9; the price of DG as well as feed decrease from Figure 5.10; and the quantity of DG as well as feed increase from Figure 5.11.

a) When $\tau_f \rightarrow 1$, there is the lowest feed price, thus the quantity of feed achieves the highest amount according to the down-slope demand function of feed. The more DG in the feed is, the more profit of both producers gains.

b) When $\tau_f \rightarrow 0$, there is the highest feed price $\$175/\text{ton}$ which is less than the maximum feed price $\alpha_{df} = \$180/\text{ton}$, thus the quantity of feed achieves the lowest amount according to the down-slope demand function of feed. At the same time, the DG price is approaching infinite because the feed producer utilizes extremely small amount of DG, which is close to zero for the feed production. The feed producer has the profit from selling $100,000\text{ton}$ of feed and the E&DG producer has the profit by selling extremely small amount of DG with the extremely high DG price.

And usually the DG fraction for all kinds of animal has been ranged in (0,0.6] [50],[51],[52],[53]. So, the E&DG producer has $\prod_E{}^{ed*}$ ranged in $(\$1,000,000, \$7,840,000]$, and the feed producer has $\prod_F{}^{ed*}$ ranged in $(\$500,000,\$3,920,000]$. The DG price is ranged in $[\$136/\text{ton},\infty)$ and the feed price is ranged in $[\$166/\text{ton},\$175/\text{ton})$. And the DG quantity is ranged in $(0\text{ton},168,000\text{ton}]$ and the feed quantity is ranged in $(100,000\text{ton},280,000\text{ton}]$.

When given $\tau_f = 0.40$, the E&DG producer has $\prod_E{}^{ed*} = \$4,840,000$ and the feed producer has $\prod_F{}^{ed*} = \$2,420,000$. The DG price is $p_d{}^{ed*} = \$145/\text{ton}$ and the feed price is $p_{df}{}^{ed*} = \$169/\text{ton}$. In addition, the DG quantity is $D_d{}^{ed*} = 88,000\text{ton}$ and the feed quantity is $D_{df}{}^{ed*} = 220,000\text{ton}$.

With the lowest DG fraction $\tau_f \to 1$, both producers have the maximum profit respectively and the quantity of feed is in the maximum since the feed price is the lowest. And more, the DG price is the lowest so that the quantity of DG as well as ethanol is the highest, which is helpful for the expanding ethanol market. Under the condition in Case 3, with the highest DG fraction, both producers have more profit and the E&DG producer produces the maximum quantity of ethanol.

Since in Case 3 the increase of the DG fraction helps the expanding ethanol market and increases the profit of both producers. Next, we want to present the numerical solutions of both the ED model and the CC model in Case 3 with the other ingredients' cost $c_f = \$150/\text{ton}$

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5.2 Numerical solution of the ED model

$$k_e = k_d = 1/3, \alpha_{df} = 180, \beta_{df} = 5\times10^{-5}, \tau_f = 0.40,$$
$$p_e = \$750/\text{ton}, c_{em} = \$80/\text{ton}, c_c = \$180/\text{ton}, c_{dd} = \$60/\text{ton}, c_{fm} = \$10/\text{ton}, c_f = \$150/\text{ton}.$$

According to Eq. (2.47), this numerical example is in the Case 3 since $\mathrm{A} < c_f k_d$ for $\mathrm{A} = (c_{em} + c_c + c_{dd} k_d) - k_e p_e = 30$ and $c_f k_d = 50$.

Figure 5.12 shows that the E&DG producer maximizes his profit by controlling the price of DG p_d as Eq. (2.11), while he knows the best response function of the feed producer is $p_{df}(p_d)$ in Eq. (2.15). Therefore, the peak in Figure 5.12 is that the equilibrium p_d^{ed*} for DG is \$145/ton and $\prod_E^{ed*}$ is \$4,840,000.

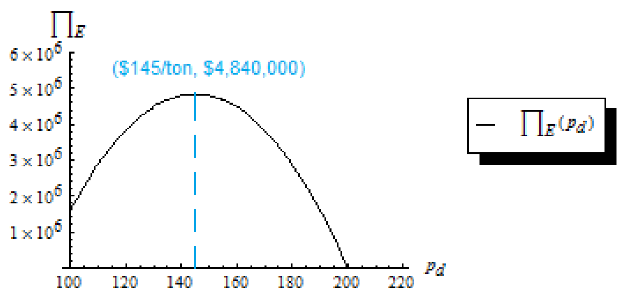

Figure 5.12 The profit of the E&DG producer w.r.t. the price of DG in the ED model

Similarly, Figure 5.13 shows that the feed producer in the ED model maximizes her own profit by controlling the price of feed p_{df} as Eq. (11), while $p_d^{ed*} = \$145/\text{ton}$ is given by the E&DG producer. Therefore, the peak in Figure 5.13 is that p_{df}^{ed*} is \$169/ton and $\prod_F^{ed*}$ is \$2,420,000.

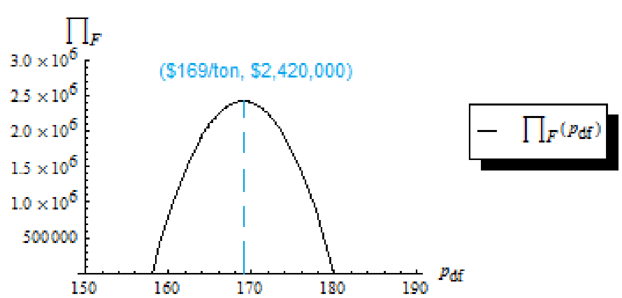

Figure 5.13 The profit of the feed producer w.r.t. the feed price in the ED model

Therefore, from Figure 5.12 and Figure 5.13, the equilibrium prices in DG as well as feed are $\{p_d^{ed*} = \$145/\text{ton}, p_{df}^{ed*} = \$169/\text{ton}\}$. The quantity of feed is $D_{df}^{ed*} = 220,000\text{ton}$ according to the assumption **A1**. The profit of the E&DG producer is $\prod_E^{ed*} = \$4,840,000$, and the profit of the feed producer is $\prod_F^{ed*} = \$2,420,000$ as shown in Figure 5.12 and Figure 5.13. At the equilibrium point, the E&DG producer has twice more profit than the feed producer, $\prod_E^{ed*} = 2\prod_F^{ed*}$.

5.3 Comparison among the ED model and the CC model

Table 5.1 Numerical values of the ED model and the CC model

	D_{df}^* (ton)	D_d^* (ton)	Q^* (ton)	D_e^* (ton)	p_{df}^* (\$/ton)	p_d^* (\$/ton)	$\prod_E^*$ (\$)	$\prod_F^*$ (\$)	$\prod_{CC}^*$ (\$)
ED	220,000	88,000	264,000	88,000	169	145	4,840,000	2,420,000	7,260,000
CC	440,000	176,000	528,000	176,000	158	NA	N/A	N/A	9,680,000

In the CC model, the optimal feed price is $p_{df}^{cc*} = \$158/\text{ton}$. When compared to the ED model, from Table 5.1, the CC model has the lower feed price along with the higher quantity of feed. In addition, the CC model has the quantity of corn, the quantity of DG and the quantity of ethanol twice more than the ED model. Therefore, more ethanol produced by the E&DG producer is provided for the expanding ethanol market.

$$p_{df}^{cc*} = \$158/\text{ton} < p_{df}^{ed*} = \$169/\text{ton}$$
$$D_{df}^{cc*} = 440,000\text{ton} = 2D_{df}^{ed*}$$
$$D_d^{cc*} = 176,000\text{ton} = 2D_d^{ed*}$$
$$Q^{cc*} = 528,000\text{ton} = 2Q^{ed*}$$
$$D_e^{cc*} = 176,000\text{ton} = 2D_e^{ed*}$$

By coordination, the total channel profit under the CC model is higher than the total profit in the ED model. Government might want to facilitate the sharing of the supply chain profit by subsidy, etc. to promote the higher production of ethanol.

$\prod_{CC}{}^{*} = TR^{cc*} - TC^{cc*} = \$9,680,000 > \prod_{E}{}^{ed*} + \prod_{F}{}^{ed*} = TR^{ed*} - TC^{ed*} = \$7,260,000$, since the increased total revenue is higher than the increased total cost when the CC model is compared to the ED model, where the total revenue in the CC model is higher than that in the ED model, $\Delta TR = TR^{cc*} - TR^{ed*} > 0$, and the total cost in the CC model is higher than that in the ED model, $\Delta TC = TC^{cc*} - TC^{ed*} > 0$, from Table 5.2.

$$TR^{cc*} = R_e{}^{cc*} + R_{df}{}^{cc*} = \$201,520,000$$
$$TR^{ed*} = R_e{}^{ed*} + R_{df}{}^{ed*} = \$103,180,000$$
$$TC^{cc*} = \left(c_{em} + c_c + k_d c_{dd}\right)Q^{cc*} + c_f\left(D_{df}{}^{cc*} - D_d{}^{cc*}\right) + c_{db}D_{df}{}^{cc*} = \$191,840,000$$
$$TC^{ed*} = \left(c_{em} + c_c + k_d c_{dd}\right)Q^{ed*} + c_f\left(D_{df}{}^{ed*} - D_d{}^{ed*}\right) + c_{db}D_{df}{}^{ed*} = \$95,920,000$$

Thus, the total profit in the ED model is 3/4 of that in the CC model,

$\left(\prod_{E}{}^{ed*} + \prod_{F}{}^{ed*}\right) / \prod_{CC}{}^{*} = 0.75$.

Table 5.2 Revenues and costs in the ED model and the CC model

	$R_e{}^{*}$ (\$)	$R_d{}^{*}$ (\$)	$(c_{em} + c_c + k_d c_{dd})Q^{*}$ (\$)	$R_{df}{}^{*}$ (\$)	$c_f\left(D_{df}{}^{*} - D_d{}^{*}\right)$ (\$)	$c_{fm}D_{df}{}^{*}$ (\$)
ED	66,000,000	12,760,000	73,920,000	37,180,000	19,800,000	2,200,000
CC	132,000,000	NA	147,840,000	69,520,000	39,600,000	4,400,000
Ratio b/w ED and CC	0.50	NA	0.50	0.53	0.50	0.50

Table 5.2 shows the revenues in selling ethanol, DG, and feed, respectively, and includes the costs for the E&DG production, other ingredients, and feed production, respectively.

Compared with the ED model, the CC model will gain more revenue in selling ethanol because more ethanol is produced, and gain more revenue in selling feed, although with a lower feed price. However, the CC model costs more for E&DG production, the other ingredients and the feed production with higher corresponding quantities.

$$R_e^{cc*} = \$132,000,000 = 2R_e^{ed*}$$
$$R_{df}^{cc*} = \$69,520,000 < 2R_{df}^{ed*} = \$74,360,000$$
$$\left(c_{em} + c_c + k_d c_{dd}\right)Q^{cc*} = \$147,840,000 = 2\left(c_{em} + c_c + k_d c_{dd}\right)Q^{ed*}$$
$$c_f\left(D_{df}^{cc*} - D_d^{cc*}\right) = \$39,600,000 = 2c_f\left(D_{df}^{ed*} - D_d^{ed*}\right)$$
$$c_{fm}D_{df}^{cc*} = \$4,400,000 = 2c_{fm}D_{df}^{ed*}$$

5.3.1 The analysis of supply chain models with respect to the DG fraction τ_f

When there is the other ingredients' cost $c_f = \$150/\text{ton}$, as the DG fraction increases, the supply chain profit under both models increase, and the CC model has higher supply chain profit than the ED model from Figure 5.14. When given $\tau_f = 0.40$, the supply chain profit in the CC model is $\$9,680,000$ and in the ED model is $\$7,260,000$. Moreover, usually the DG fraction for all kinds of animal has been ranged from 0 to 0.60. So, the CC model has the supply chain profit ranged in $(\$2,000,000, \$15,680,000]$, and the ED model has the supply chain profit ranged in $(\$1,500,000, \$11,760,000]$.

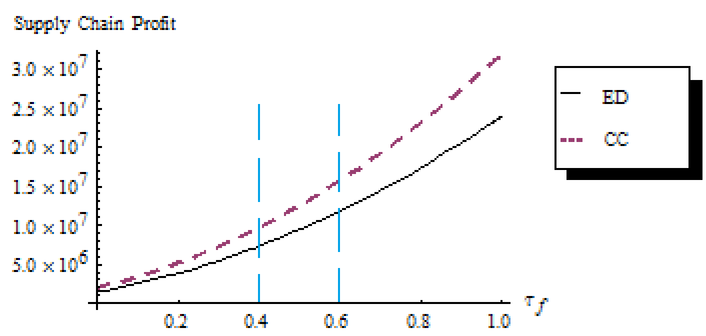

Figure 5.14 The analysis of the supply chain profit w.r.t the DG fraction

In this numerical example with $\tau_f = 0.40$, the quantity of corn used for the production under both models is $Q^{cc*} = 528,000\text{ton}$ and $Q^{ed*} = 264,000\text{ton}$, respectively. Thus, Figure 5.15 shows the quantity of corn under both models. When the DG fraction for all kinds of animal has been ranged from 0 to 0.60, the quantity of corn is ranged in $(0\text{ton}, 1,008,000\text{ton}]$ in the CC model and in $(0\text{ton}, 504,000\text{ton}]$ in the ED model. According to the assumption **A3** and Eq. (2.2), in the CC model, the quantity of DG as well as ethanol is ranged in $(0\text{ton}, 168,000\text{ton}]$, and the feed quantity is ranged in $(100,000\text{ton}, 280,000\text{ton}]$.

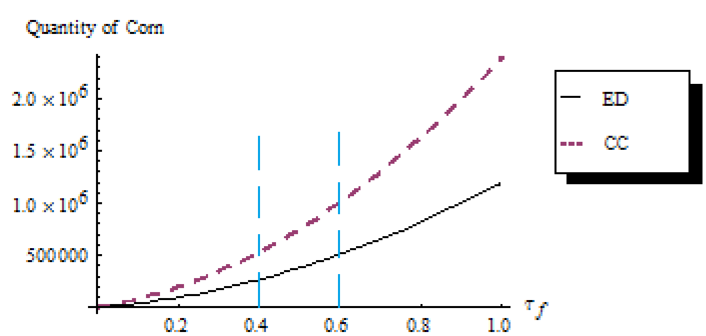

Figure 5.15 The analysis of the quantity of corn w.r.t the DG fraction

5.4 Numerical solution of the EDQ model

In summary, the parameters in the numerical example are:

$$k_e = k_d = 1/3, \alpha_{df} = 180, \beta_{df} = 5\times10^{-5}, \theta = 320, \gamma = 2\times10^{-9}, \tau_f = 0.40,$$
$$p_e = \$750/\text{ton}, c_c = \$180/\text{ton}, c_{dd} = \$60/\text{ton}, c_{fm} = \$10/\text{ton}, c_f = \$150/\text{ton}.$$

5.4.1 The unit and total joint production cost in the EDQ model

Hence, from Eq. (4.1) in the EDQ model we have a function to express the total joint production cost of the E&DG producer with regard to the quantity of corn used for the production in (5.1).

$$\hat{C}_{EM}(Q) = \theta Q - \sqrt{3\gamma\theta}Q^2 + \gamma Q^3 = 320Q - 8\sqrt{3}\times10^{-4}Q^2 + 2\times10^{-9}Q^3 \tag{5.1}$$

The E&DG producer has the unit joint production cost as Figure 5.16 and the total joint production cost as Figure 5.17. The unit joint production cost is a quadratic function with regard to the quantity of con used for the production; the total joint production cost is a monotonic increasing function as the increase of the quantity of con used for the production.

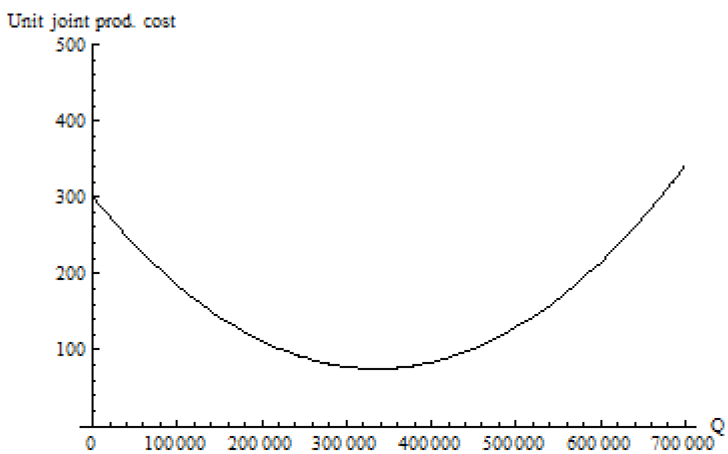

Figure 5.16 The unit joint production cost of the E&DG producer

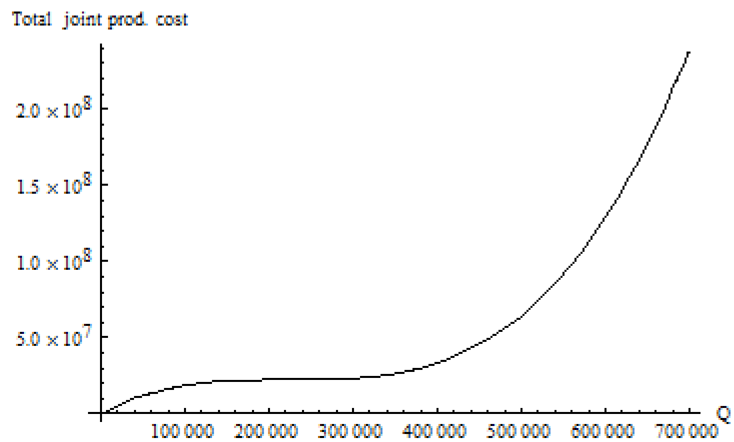

Figure 5.17 The total joint production cost of the E&DG producer

5.4.2 Numerical example

This numerical example satisfies $\mathrm{K}-2\Gamma>0$ in Eq. (4.12) and $\mathrm{K}+2\Gamma>0$ in Eq. (4.13), so we have the equilibrium solution listed in Table 4.1. The numerical values of each variables in the EDQ model are listed in Table 5.3.

Table 5.3 Numerical values of the ED model and the EDQ model

	D_{df}^* (ton)	D_d^* (ton)	Q^* (ton)	D_e^* (ton)	p_{df}^* ($/ton)	p_d^* ($/ton)	Π_E^* ($)	Π_F^* ($)	Π_{CC}^* ($)
ED	220,000	88,000	264,000	88,000	169	145	4,840,000	2,420,000	7,260,000
EDQ	282,222	112,889	338,666	112,889	166	130	4,412,230	3,982,460	8,394,690

From Table 5.3, compared to the EDQ model, the ED model has the lower quantity in feed, DG, corn, as well as ethanol, has the higher price in feed as well as DG, and has the higher profit for the E&DG producer, the lower profit for the feed producer, and the lower profit for the total profit of the supply chain.

Figure 5.18 presents the total joint production cost of the E&DG producer under the ED model as well the EDQ model. Since $c_{em}=\theta/4=80$, Figure 5.18 is in the situation 1.

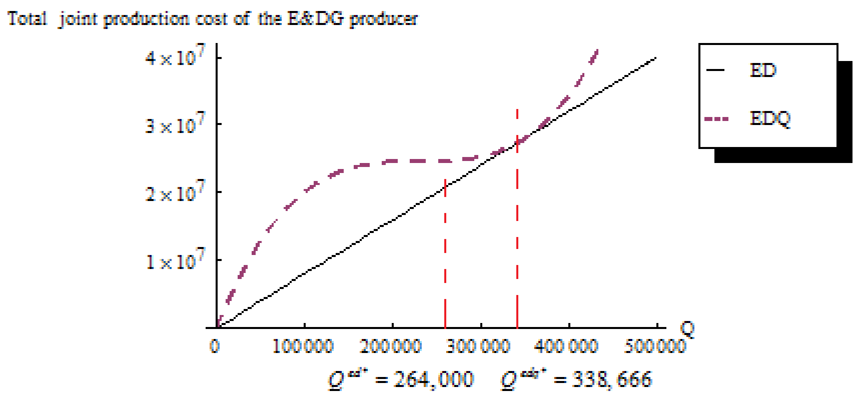

Figure 5.18 The total joint production cost of the E&DG producer in the ED & EDQ models

$$c_{em}Q^{ed*} = \$2.112 \times 10^{7}$$
$$\hat{C}_{EM}(Q^{edq*}) = \theta Q^{edq*} - \sqrt{3\gamma\theta}\left(Q^{edq*}\right)^{2} + \gamma\left(Q^{edq*}\right)^{3} = \$2.71339 \times 10^{7}$$

The EDQ model spends more joint production cost than the ED model from $c_{em}Q^{ed*} < \hat{C}_{EM}(Q^{edq*})$.

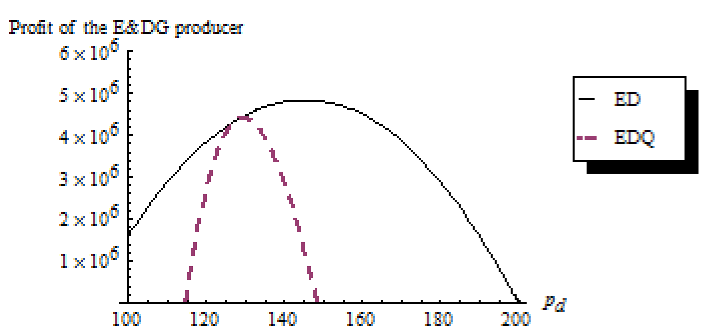

Figure 5.19 The profit of the feed producer in the ED model as well as the EDQ model

Figure 5.20 shows that the E&DG producer maximizes his profit by controlling p_d as Eq. (2.11) in the ED model and Eq.(4.8) in the EDQ model, while he knows the best response function for the feed producer is $p_{df}(p_d)$ in Eqs. (2.15) and (4.5). Therefore, the equilibrium

p_d^{ed*} in the ED model is \$145/ton and $\prod_E^{ed*}$ is \$4,840,000, and the equilibrium p_d^{edq*} in the ED model is \$130/ton and $\prod_E^{edq*}$ is \$4,412,230.

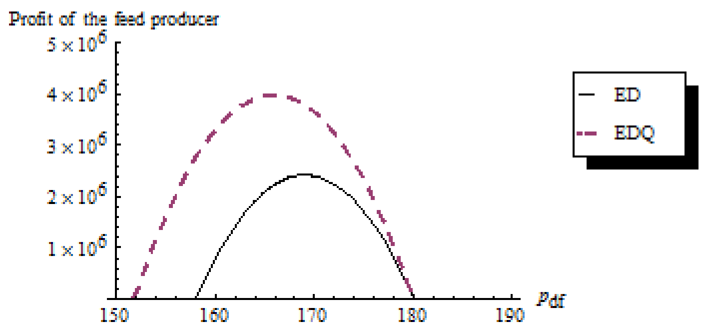

Figure 5.20 The profit of the E&DG producer in the ED model as well as the EDQ model

Similarly, Figure 5.20 shows that, the feed producer in the ED model maximizes her own profit by controlling p_{df} as Eq. (2.12) in the ED model and Eq.(4.4) in the EDQ model, while $p_d^{ed*} = \$145/\text{ton}$ and $p_d^{edq*} = \$130/\text{ton}$ are given by the E&DG producer. Therefore, p_{df}^{ed*} is \$169/ton and $\prod_F^{ed*}$ is \$2,420,000, and the equilibrium p_{df}^{edq*} in the ED model is \$166/ton and $\prod_F^{edq*}$ is \$3,982,460.

6. CONCLUDING REMARKS AND FUTURE WORK

Many studies about the competitive or coordination relationship between two successive producers only study for one final product. Instead, our paper extends to study the Stackelberg competition between two successive producers: the E&DG producer as the Stackelberg leader produces two joint output products (ethanol and DG), and the feed producer as the Stackelberg follower utilizes DG for feed production. Then the equilibrium consequences are explored in terms of profits, prices, and demands, which shows the E&DG producer gains more profit than the feed producer does. After analyzing with respect to parameters from the feed producer, i.e., the DG fraction, the cost of other ingredients, the E&DG producer has more sensitive profit to the change in these parameters than the feed producer does. When other ingredients' cost increases, the profit of both producers will lose profit and the quantity of DG as well as ethanol decreases, since the increasing cost to produce feed results in the increase of the feed price. However, the increase of the DG fraction under specific cases helps to increase the quantity of DG as well as ethanol in order to help the expanding ethanol market.

Being a group, the CC model is the coordination scenario to compare the Stackelberg model with respect to the supply chain profit and the performance. Compared with the Stackelberg model, the centrally coordinated model has higher total profit to be shared by both producers, and has higher quantity of ethanol provided to the quick expanding ethanol market.

In the part of work in progress, 1) the supply chain with a revenue sharing contract is introduced for the higher coordinated profit. 2) the supply of DG from the E&DG is greater than its demand from the feed producer, and the surplus of DG is costly disposed of. And the E&DG producer has the capacity constraint for his production. With these constraints, we explored under what condition for the cost of discarding, the E&DG producer would not like to discard.

The analysis presented in this paper leads to several interesting areas for further research: 1) When most of other ingredient in feed is corn, what is the impact of the DG fraction when the corn price is changing. 2) The effort of the feed producer to consume more DG is taken into account. $\tau_f = a - b \times p_d / c_c$, the DG fraction in feed is the down slope function with regard to the ratio between the DG price and the corn price [56]. Under a certain condition, the incentive will lose effectiveness, and the E&DG producer will reject it. 3) When the surplus of DG occurs, the E&DG producer will select the quantity discount to sell them. Then, we can study how the quantity discount strategy is applied in the supply chain.

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